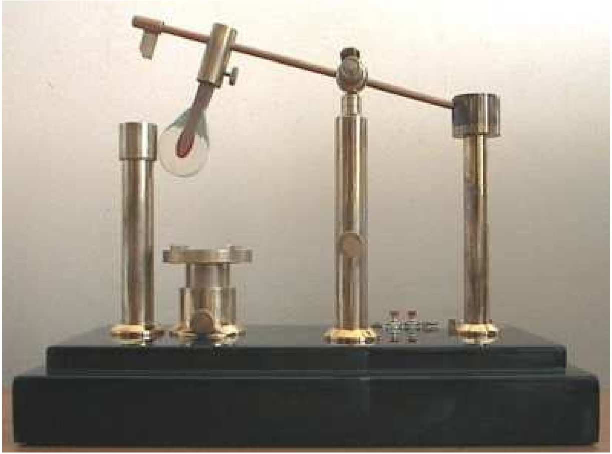
Acoustical properties of the wool felt

Anatoli Stulov & Dmitri Kartofelev

Institute of Cybernetics at Tallinn University of Technology

Piano hammers

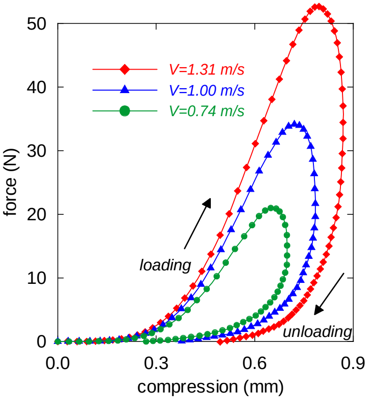
Piano hammer testing device



Experimental results

Piano hammer features

- *The relationships of dynamic force versus felt compression show the significant influence of hysteresis characteristics, so the loading and unloading of the felt are not alike*
- *Force-compression characteristics of the hammer felt are essentially nonlinear*
- *The slope of the dynamic force-compression characteristics is strongly dependent on the rate of loading*



Hysteretic Model of Piano Hammer

(four-parameter model)

$$F(u) = Ku^p$$

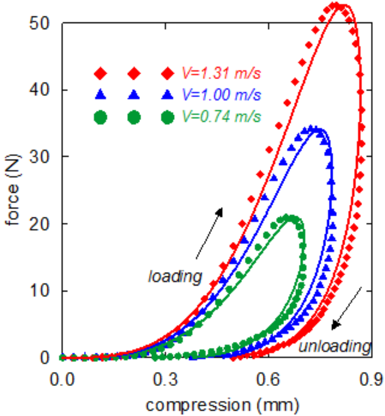
$$K \Rightarrow F_0[1 - R(t)\,*]\,, \qquad R(t) = \frac{\varepsilon}{\tau_0}\exp\left(-\frac{t}{\tau_0}\right)$$

$$F(u(t)) = F_0\left[u^p(t) - \frac{\varepsilon}{\tau_0}\int_0^t u^p(\xi)\exp\left(\frac{\xi - t}{\tau_0}\right)d\xi\right]$$

Numerical simulation of experiment

$$m\frac{d^2u}{dt^2} = F_0\left[u^p(t) - \frac{\varepsilon}{\tau_0}\int_0^t u^p(\xi)\exp\left(\frac{\xi - t}{\tau_0}\right)d\xi\right]$$

$$u(0) = 0,\ \frac{du}{dt}(0) = V$$

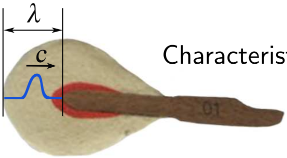


Hammer parameters

- $F_0 = 8800 N/mm^p$
- $p = 3.95$
- $\varepsilon = 0.992$
- $\tau_0 = 2.0\mu s$

$t_0 = \mathbf{1.7},\ \mathbf{2.0},\ \mathbf{2.5}\ ms$

Felt model



Characteristic time $\tau \sim 2\lambda/c$, $\quad c = \sqrt{\frac{E}{\rho}}$

$$F(u) = F_0\left[u^p(t) - \mathcal{K}(t) * u^p(t)\right], \quad \mathcal{K}(t) = \frac{\varepsilon}{\tau_0}\exp\left(-\frac{t}{\tau_0}\right)$$

$$\Downarrow$$

$$\sigma(\epsilon) = E_d\left[\epsilon^p(t) - \mathcal{K}(t) * \epsilon^p(t)\right], \quad \mathcal{K}(t) = \frac{\varepsilon}{\tau_0}\exp\left(-\frac{t}{\tau_0}\right)$$

here: σ – stress, $\epsilon = u_x$ – strain, u – displacement
F_0 is the instantaneous hammer stiffness
E_d – dynamic Young's modulus

Wave propagation in felt

Equation of motion

$$\rho u_{tt}(x,t) - \sigma_x(x,t) = 0$$

Initial conditions

$$u(x,0) = u_t(x,0) = \sigma(x,0) = 0$$

Constitutive equation

$$\sigma(\epsilon) = E_d[\epsilon^p(t) - \mathcal{K}(t) * \epsilon^p(t)] \,, \quad \mathcal{K}(t) = \frac{\varepsilon}{\tau_0} \exp\left(-\frac{t}{\tau_0}\right)$$

σ – stress, $\epsilon = u_x$ – strain, u – displacement

Equations and parameters

$$u \Rightarrow u/l, \quad x \Rightarrow x/l_0, \quad t \Rightarrow t/\alpha_0$$

$$[(u_x)^p]_x - u_{tt} + [(u_x)^p]_{xt} - \delta u_{ttt} = 0, \quad u - \text{displacement}$$

$$[\epsilon^p]_{xx} - \epsilon_{tt} + [\epsilon^p]_{xxt} - \delta \epsilon_{ttt} = 0, \quad \epsilon = u_x - \text{strain}$$

$$\alpha_0 = \tau_0/\delta,\ l_0 = c_d \alpha_0 \sqrt{\delta},\ \delta = 1 - \varepsilon,\ E_s = \delta E_d,\ c_s = c_d \sqrt{\delta}$$

ε=0.99, δ=0.01, τ_0=20μs, $p \sim 2$

ρ=10^3kg/m^3, E_d=60MPa, E_s=0.6MPa, c_s=25m/s, c_d=250m/s

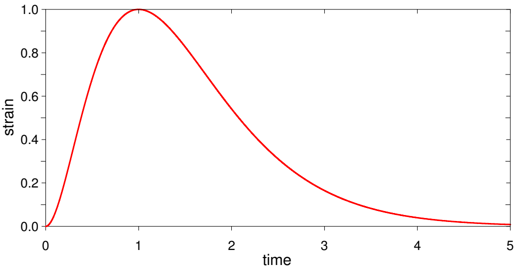
1[x]=l_0=50mm , 1[t]=α_0=2ms

Boundary value problem

$$[\epsilon^p]_{xx} - \epsilon_{tt} + [\epsilon^p]_{xxt} - \delta\epsilon_{ttt} = 0$$

$$\begin{cases} z = \epsilon^p \\ \epsilon_{tt} = w_{xx} \\ w = z - \varepsilon \int_0^t z \exp(\xi - t) d\xi \end{cases}$$

$$\epsilon(x,0) = \epsilon_t(x,0) = 0$$



$$\epsilon(0,t) = f(t) = \left(\frac{t}{t_0}\right)^2 \exp[2(1-t/t_0)]$$

Linear case (p=1) and dispersion equation

$$(\epsilon^p)_{xx} - \epsilon_{tt} + (\epsilon^p)_{xxt} - \delta\epsilon_{ttt} = 0$$

$$\Downarrow$$

$$\epsilon_{xx} - \epsilon_{tt} + \epsilon_{xxt} - \delta\epsilon_{ttt} = 0$$

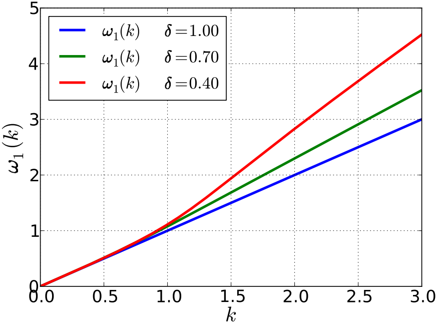
$$\Downarrow$$

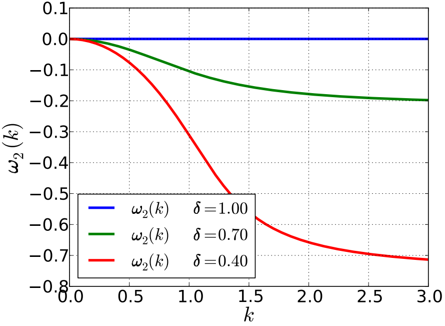
$$k^2 - \omega^2 - i\omega k^2 + i\delta\omega^3 = 0$$

$$\omega = \omega(k) = \omega_1 + i\omega_2$$

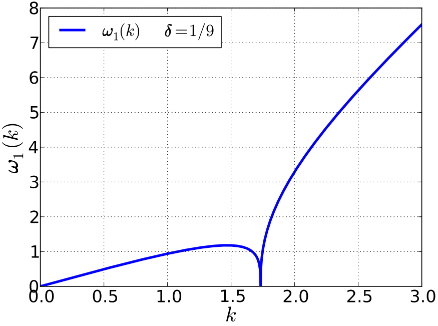
$$\omega_1 = Re\,\omega(k), \;\; \omega_2 = Im\,\omega(k)$$

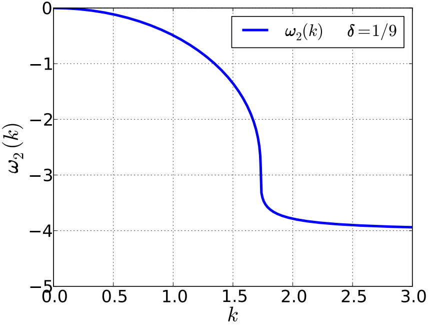
Dispersion curves ($\delta > 1/9$)



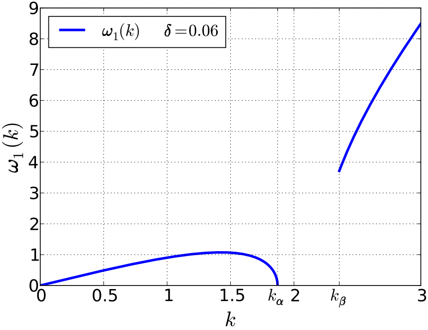


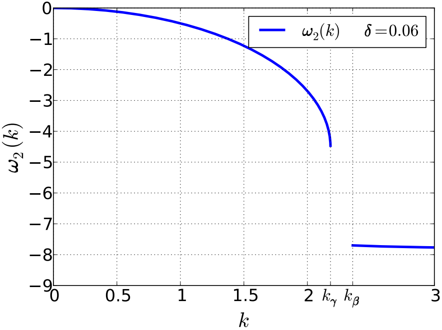
Dispersion curves ($\delta = 1/9$)



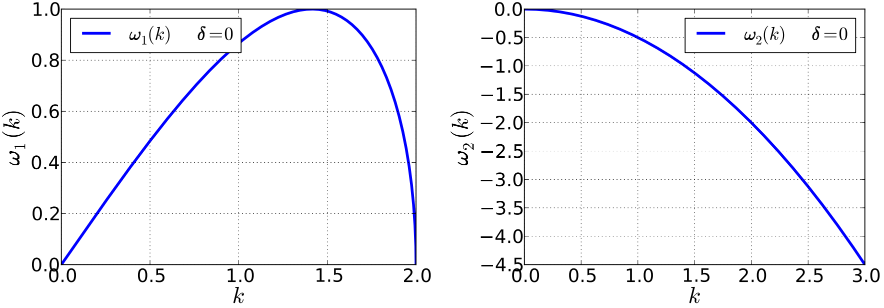


Dispersion curves ($\delta < 1/9$)





Dispersion curves ($\delta = 0$)



Strain wave propagation

(Boundary value problem)

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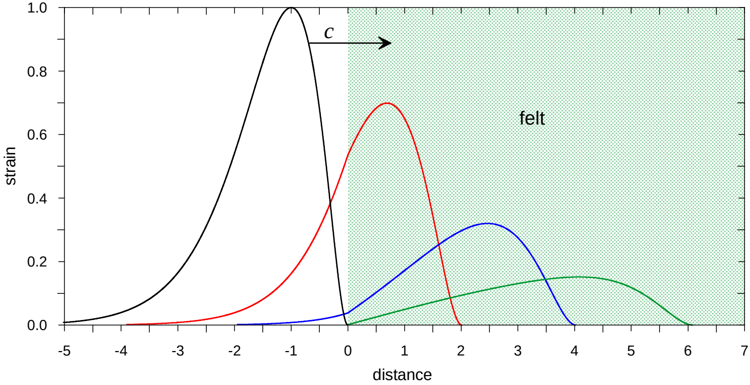
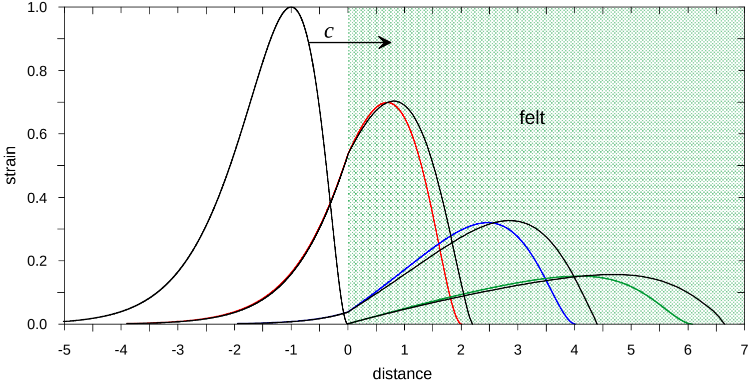


Through the linear felt

Strain wave propagation

(Boundary value problem)



Through the linear and nonlinear felt

Strain wave propagation

(Initial value problem)

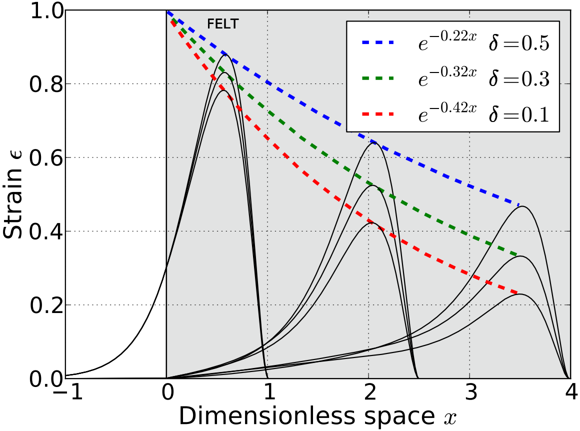
Strain wave propagation

(Initial value problem)

Strain wave propagation

(Initial value problem)

Wave attenuation



Summary

- Equations that describe the strain wave propagation in the nonlinear hereditary felt are derived using the experimental data from the piano hammer studies
- These equations are solved numerically by using the explicit finite differences method
- A bell-like pulse propagation is considered, and the influence of values of parameters p and δ on the changing of the pulse form is analyzed
- The rate of the wave attenuation in the felt material is also estimated