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Wave propagation and dispersion in fluid-saturated rigid-framed porous media: checking a new nonlocal macroscopic theory

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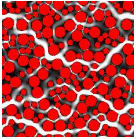
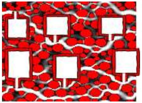
Outline

- Introduction
- From microscopic to macroscopic
- Macroscopic equations: local and nonlocal
- Procedure for computing effective density and compressibility
- Checking the nonlocal theory

A macroscopic theory describing the long-wavelegth acoustic propagation through porous media

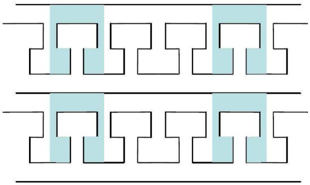
- Saturated porous media: fluid-solid
  - Solid is rigid
  - Fluid is visco-thermal
- Local theory (Classical Equivalent-Fluid = order "0" of the Homogenization Theory=temporal dispersion)
  - Viscous dissipation
  - Thermal dissipation
  - Microscopic scale: the fluid is incompressible $\rightsquigarrow$ $\nabla.\mathbf{v} = 0$
  - Local theory is not complete...





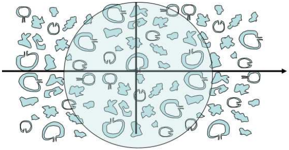
Generalization

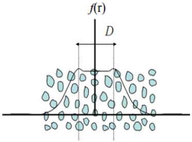
- Nonlocal theory
  - Temporal dispersion
    AND
  - Nonlocal effects due to spatial dispersion
  - Viscous and thermal dissipation
  - Microscopic scale: the fluid is compressible $\rightsquigarrow \nabla.\mathbf{v} \neq 0$
  - Porous medium is isotropic or having a prefered axis



From Microscopic to Macroscopic

- Microscopic scale: Navier-Stokes-Fourier
- D: averaging scale=REV λ: wavelength $D \ll \lambda$
- Indicator function: $I(\mathbf{r}) = \begin{cases} 1, & \mathbf{r} \in \mathcal{V}^f \text{ fluid region} \\ 0, & \mathbf{r} \in \mathcal{V}^s \text{ solid region} \end{cases}$
- Definition $\langle \mathbf{v}(\mathbf{r}, t) \rangle = \int_{\mathcal{V}} d^3r' \; f(\mathbf{r}' - \mathbf{r}) I(\mathbf{r}') \mathbf{v}(\mathbf{r}', t)$
  $\langle p(\mathbf{r}, t) \rangle = \int_{\mathcal{V}} d^3r' \; f(\mathbf{r}' - \mathbf{r}) I(\mathbf{r}') p(\mathbf{r}', t)$





- Local theory: macroscopic velocity $\mathbf{V}(\mathbf{r}, t) = \langle \mathbf{v}(\mathbf{r}, t) \rangle$
  macroscopic pressure $P(\mathbf{r}, t) = \langle p(\mathbf{r}, t) \rangle$
- Nonlocal theory: macroscopic velocity $\mathbf{V}(\mathbf{r}, t) = \langle \mathbf{v}(\mathbf{r}, t) \rangle$
  macroscopic pressure $\langle p(\mathbf{r}, t) \mathbf{v}(\mathbf{r}, t) \rangle = P(\mathbf{r}, t) \langle \mathbf{v}(\mathbf{r}, t) \rangle$

Macroscopic equations: local and nonlocal

- Local theory: pressure and velocity satisfy

$$\hat{\rho}\frac{\partial \mathbf{V}}{\partial t} = -\nabla P, \qquad \hat{\chi}\frac{\partial P}{\partial t} = -\nabla . \mathbf{V}$$

where

$$\hat{\rho}\mathbf{V}(\mathbf{r}, t) = \int_0^\infty dt' \; \rho(t - t')\mathbf{V}(\mathbf{r}, t')$$
$$\hat{\chi}P(\mathbf{r}, t) = \int_0^\infty dt' \; \chi(t - t')P(\mathbf{r}, t')$$

- Nonlocal theory: pressure and velocity satisfy

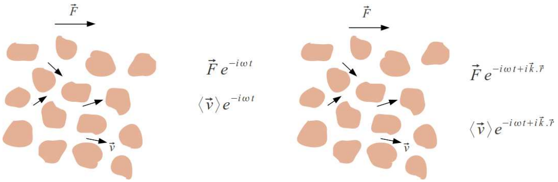
$$\hat{\rho}\frac{\partial \mathbf{V}}{\partial t} = -\nabla P, \qquad \hat{\chi}\frac{\partial P}{\partial t} = -\nabla . \mathbf{V}$$

where

$$\hat{\rho}\mathbf{V}(\mathbf{r}, t) = \int_0^\infty dt' \int_{\mathcal{V}} d^3r' \; \rho(t - t', \mathbf{r} - \mathbf{r}')\mathbf{V}(\mathbf{r}', t')$$
$$\hat{\chi}P(\mathbf{r}, t) = \int_0^\infty dt' \int_{\mathcal{V}} d^3r' \; \chi(t - t', \mathbf{r} - \mathbf{r}')P(\mathbf{r}', t')$$

Procedure to dermine effective density and compressibility

How to obtain $\rho(\omega)$, $\chi(\omega)$, $\rho(\mathbf{k},\omega)$ and $\chi(\mathbf{k},\omega)$ from microscale?



They are determined by solving two independent problems:

1- Response of the fluid subjected to a bulk force
- time varying $\rightsquigarrow$ $\rho(\omega)$
- time and spatial varying $\rightsquigarrow$ $\rho(k,\omega)$

2- Response of the fluid subjected to a heating source
- time varying $\rightsquigarrow$ $\chi(\omega)$
- time and spatial varying $\rightsquigarrow$ $\chi(k,\omega)$

$$-i\omega\rho(k,\omega) = -ikP(k,\omega), \qquad -i\omega\chi(k,\omega) = -ikV(k,\omega)$$

Procedure to determine effective density

- In the visco-thermal fluid

$$\frac{1}{\rho_0}\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{v} = 0$$

$$\rho_0 \frac{\partial \mathbf{v}}{\partial t} = -\nabla p + \eta \nabla^2 \mathbf{v} + \left(\zeta + \tfrac{1}{3}\eta\right) \nabla \left(\nabla . \mathbf{v}\right) + \mathbf{F} e^{ikx - i\omega t}$$

$$\rho_0 c_p \frac{\partial \tau}{\partial t} = \beta_0 T_0 \frac{\partial p}{\partial t} + \kappa \nabla^2 \tau$$

$$\gamma \chi_0 p = \frac{\rho}{\rho_0} + \beta_0 \tau$$

- On the fluid-solid interface

$$\mathbf{v} = 0, \quad \tau = 0$$

where $\mathbf{F} e^{ikx - i\omega t} = -\nabla \left(\Phi e^{ikx - i\omega t}\right)$

$$\rho(k, \omega) = \frac{k\left(\Phi + P(k, \omega)\right)}{\omega \left\langle v(\mathbf{r}, k, \omega)\right\rangle}$$

Procedure to determine effective compressibility

- In the visco-thermal fluid

$$\frac{1}{\rho_0}\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{v} = 0$$

$$\rho_0 \frac{\partial \mathbf{v}}{\partial t} = -\nabla p + \eta \nabla^2 \mathbf{v} + \left(\zeta + \tfrac{1}{3}\eta\right) \nabla \left(\nabla . \mathbf{v}\right)$$

$$\rho_0 c_p \frac{\partial \tau}{\partial t} = \beta_0 T_0 \frac{\partial p}{\partial t} + \kappa \nabla^2 \tau + \beta_0 T_0 Q e^{ikx - i\omega t}$$

$$\gamma \chi_0 p = \frac{\rho}{\rho_0} + \beta_0 \tau$$

- On the fluid-solid interface

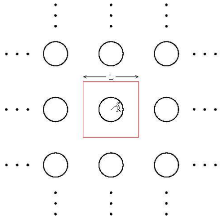
$$\mathbf{v} = 0, \quad \tau = 0$$

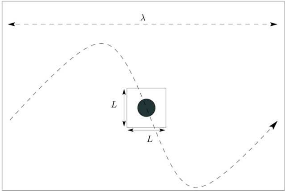
where $Q e^{ikx - i\omega t} = -\frac{\partial}{\partial t}\left(\Phi e^{ikx - i\omega t}\right)$

$$\chi(k, \omega) = \frac{\rho_0^{-1} \langle \rho(\mathbf{r}, k, \omega) \rangle + \chi_0 \gamma \Phi}{\Phi + P}$$

Finite Element simulations

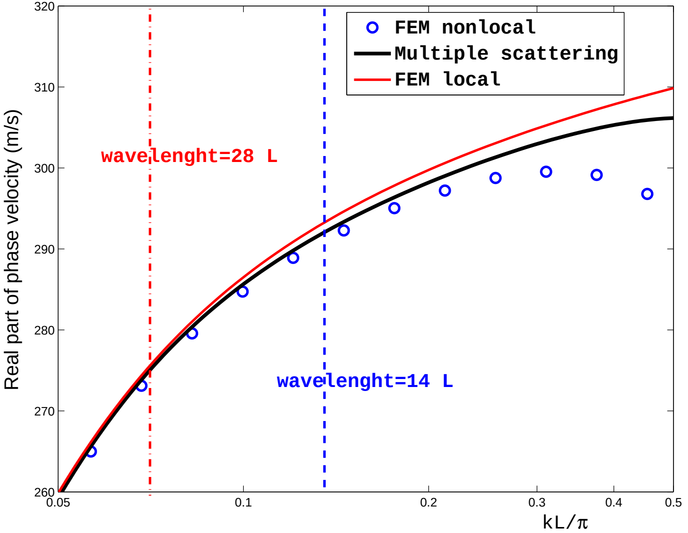
- Solving the microscopic equations with the geometry: 2D arrays of rigid cylinders- Finite Element FreeFem++





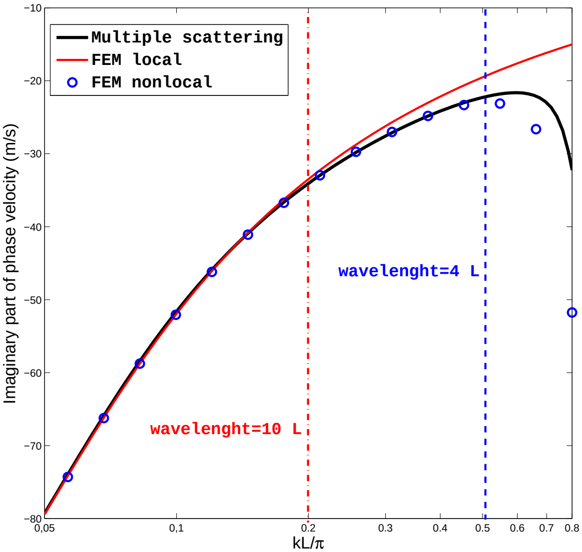
  - Local theory: phase velocity of the unic wave
    $$c(\omega) = \frac{1}{\sqrt{\rho(\omega)\chi(\omega)}}$$
  - Nonlocal theory: may be more than one wave solutions of the dispersion equation $\rho(k,\omega)\chi(k,\omega)\omega^2/k^2 = 1$
    phase velocity of the least attenuated wave $c(\omega) = \dfrac{\omega}{k}$
  - Multiple scatering (quasi-exact): phase velocity of the least attenuated wave (Duclos et al., 2009)

Finite Element Simulations: Results



$L = 10\ \mu,\ R = 1.8\ \mu,\ \phi = 0.9$

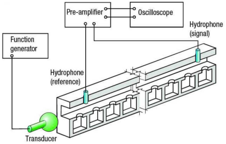
Finite Element Simulations: Results



$L = 10\ \mu,\ R = 1.8\ \mu,\ \phi = 0.9$

Conclusions and Perspectives

- A procedure to compute the effective density and compressibility for a new nonlocal equivalent-fluid theory has been proposed
- Validation of the nonlocal theory has been performed in a particular geometry for which the results from multiple scattering method are available
- Possibility that in certain geometries the nonlocal theory leads to new behaviors
- Comparison with the higher order of homogeneization theory
- Geometries leading to spatial dispersion for developing more efficient sound absorbing materials



N. Fang et al., Ultrasonic metamaterials with negative modulus, Nature, 5 (2006)

References

- D. Lafarge, The equivalent fluid model, in Materials and Acoustics Handbook, ISTE-Wiley, London, 2009, pp 205-228.
- D. Lafarge, N. Nemati, Theory of wave propagation and dispersion in fluid-saturated porous media allowing for spatial dispersion, Days on Diffraction – 2011, 30 May – 3 June, 2011, Saint-Petersburg
- A. Duclos, D. Lafarge, V. Pagneux, Transmission of acoustic waves through 2D phononic crystal: visco-thermal and multiple scattering effects, Eur. Phys. J. Appl. Phys. 45 (2009)
- C. Boutin. Rayleigh scattering of acoustic waves in rigid porous media. J. Acoust. Soc. Am., 122 :1888-1905.
- N. Fang et al., Ultrasonic metamaterials with negative modulus, Nature, 5 (2006)
- http://www.freefem.org/ff++/ftp/freefem++doc.pdf