EFFICIENT HYBRID FE-TMM MODELING FOR THE VIBROACOUSTIC RESPONSE OF STRUCTURES WITH ATTACHED SOUND PACKAGES

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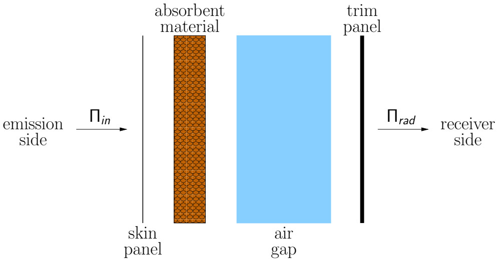
SAPEM 2011, Ferrara, Italy
14-16th December 2011

Summary

Introduction

transmission loss (TL) of double-wall (DWL) systems



- **mid/high frequencies:** the system can be assumed flat and infinitely extended → analytical wave based tools (e.g. Transfer Matrix Method, TMM) provide a good TL estimation.
- **low frequencies:** the boundary conditions must be accounted for → the full Finite Element (FE) approach is necessary.

Motivations

low frequency issues

FE modeling of poroelastic materials is computationally expensive, due to:

- frequency dependance of the matrices
- high number of DOFs per node in the fully coupled Biot's model (4 DOFs for the (u, p) formulation)
- high number of elements required in the thickness direction to capture dissipative and coupling effects

So far...

- different Biot formulations,
- substructuring + modal synthesis,
- hierarchical elements

may not be the most efficient solution.

Possible solution

Hybrid FE-TMM. Open questions:

- flat and curved DWL systems
- mechanical excitations
- active/passive inclusions

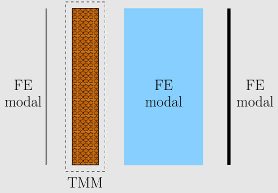
Objectives

in this work ...

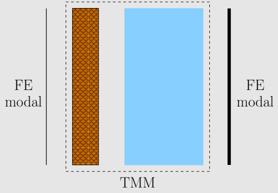
Scientific question

Considering a representative flat DWL system excited by plane waves, can we avoid the FE modeling of some sound package (SP) parts (i.e. poroelastic material, air gap and trim plate) while preserving sufficient accuracy on the TL in the low frequency range?

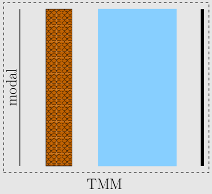
The capabilities of the following models in the low frequency range will be assessed:



Hybrid FE-TMM



Hybrid FE-TMM



Modal TMM

Considered methodologies

Reference solutions

- Full FE, direct solution (in-house code NOVAFEM)
- Full FE, substructuring (implemented in NOVAFEM)

Industrial standards

- TMM (in-house code NOVA)
- TMM with finite size corrections FTMM[1](in-house code NOVA)

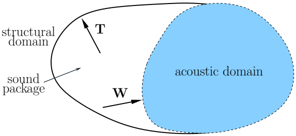
Proposed hybrid models

- Hybrid FE-TMM (implemented in NOVAFEM)
- Modal TMM (implemented in NOVA)

[1] D. Rhazi and N. Atalla. A simple method to account for size effects in the transfer matrix method. *Journal of the Acoustical Society of America*, 2010

Substructuring

Reduced order model obtained using uncoupled modes:



$$\begin{bmatrix} \hat{\mathbf{Z}} & \hat{\mathbf{C}} \\ \hat{\mathbf{C}}^{T} & \hat{\mathbf{A}} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_s \\ \mathbf{q}_a \end{Bmatrix} = \begin{Bmatrix} \hat{\mathbf{f}}_s \\ \mathbf{0} \end{Bmatrix} + \begin{Bmatrix} \hat{\mathbf{T}} \\ \hat{\mathbf{W}} \end{Bmatrix}$$

F-S Interaction

Described by modal coupling:

- $\mathbf{q}_s$ → structural modal amplitudes
- $\mathbf{q}_a$ → acoustic modal amplitudes
- $\hat{\mathbf{Z}}$ → modal impedance
- $\hat{\mathbf{A}}$ → modal admittance
- $\hat{\mathbf{C}}$ → modal coupling
- $\hat{\mathbf{f}}_s$ → structural modal load

Effect of the SP

The excitation on structural and acoustic domains due to the SP:

- $\hat{\mathbf{T}}$ → distributed surface force
- $\hat{\mathbf{W}}$ → imposed fluid displacement

can be expressed as:

$$\begin{Bmatrix} \hat{\mathbf{T}} \\ \hat{\mathbf{W}} \end{Bmatrix} = \hat{\mathbf{Y}} \begin{Bmatrix} \mathbf{q}_s \\ \mathbf{q}_a \end{Bmatrix}$$

Full FE substructuring

M.A. Hamdi *et al.*, Internoise 2000, Nice, France

Using Lagrange multipliers, the matrix $\hat{\mathbf{Y}}$ is calculated by condensation of the poroelastic DOFs:

$$\hat{\mathbf{Y}} = \mathbf{V}^{\mathrm{T}}\mathbf{C}_{fs}\left(\mathbf{C}_{p}^{\mathrm{T}}\mathbf{Z}_{p}^{-1}\mathbf{C}_{p}\right)^{-1}\mathbf{C}_{fs}^{\mathrm{T}}\mathbf{V}$$

$\mathbf{V}$ is the modal basis, $\mathbf{Z}_p$ is the poroelastic impedance, $\mathbf{C}_{fs}$ and $\mathbf{C}_p$ assure the continuity of displacements and pressure at the interface between the fluid-structure ($_{fs}$) and the poroelastic ($_p$) domains.

PROS

- exact
- $\hat{\mathbf{Y}}$ can be calculated at master frequencies and then interpolated

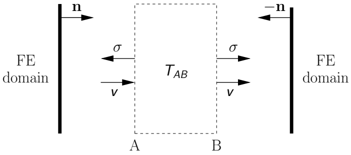
CONS

- $\hat{\mathbf{Y}}$ may exhibit peaks at some frequencies (i.e. choice of the master frequencies)
- still expensive

Hybrid FE-TMM (1)

M. Tournour *et al.*, SAE International 2007, St, Charles, Illinois

The matrix $\hat{\mathbf{Y}}$ is calculated using the Transfer Matrix of the SP:



$$\left\{ \begin{array}{c} \sigma \\ v \end{array} \right\}_A = \mathbf{T}_{AB} \left\{ \begin{array}{c} \sigma \\ v \end{array} \right\}_B$$

PROS

- a FE mesh of the entire system is not required
- $\mathbf{T}_{AB}$ evaluated by TMM
- different subsystems can be included in $\mathbf{T}_{AB}$ (e.g. air gap)
- fast

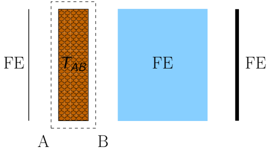
CONS

- infinite flat systems
- locally reacting behavior is assumed
- choice of the trace wavenumber k_t
- only continuity of normal stress and displacement

Hybrid FE-TMM (2)

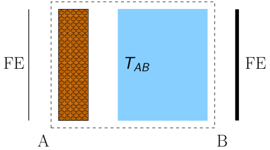
Two implementations of the FE-TMM are considered here:

- **model 1:** poroelastic → $\mathbf{T}_{AB}$
  structure and air gap → FE (modal)



$$\left\{ \begin{array}{c} \sigma^t \\ v^t \end{array} \right\}_A = \left[\begin{array}{cc} T_{11} & T_{12} \\ T_{21} & T_{22} \end{array} \right] \left\{ \begin{array}{c} \sigma^t \\ v^t \end{array} \right\}_B$$

- **model 2:** poroelastic and air gap → $\mathbf{T}_{AB}$
  structure → FE (modal)



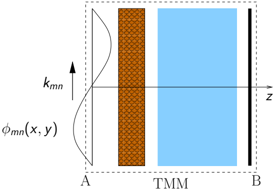
$$\left\{ \begin{array}{c} \sigma^t \\ v^t \end{array} \right\}_A = \left[\begin{array}{cc} T_{11} & T_{12} \\ T_{21} & T_{22} \end{array} \right] \left\{ \begin{array}{c} p \\ v \end{array} \right\}_B$$

Modal TMM (1)

D. Rhazi, PhD thesis, UdeS

A linear superposition of uncoupled modes is assumed

$$v(z) = \sum_{mn} v_{mn}(z)\, \phi_{mn}(x,y)$$



$$Z^A_{mn}\; v_{mn}(A) = f_{mn}$$

$$Z^B_{mn}\; v_{mn}(B) = f_{mn}$$

- v_{mn} are the modal amplitudes
- $\phi_{mn}(x,y)$ are the modal shapes of the skin panel
- f_{mn} is the modal load
- Z^A_{mn} is the impedance in A calculated by TMM for each *mn* couple
- Z^B_{mn} is the impedance in B calculated by TMM for each *mn* couple
- $k_t^2 = k_{mn}^2 = \left(\frac{m\,\pi}{a}\right)^2 + \left(\frac{n\,\pi}{b}\right)^2$ is used in the TMM

Modal TMM (2)

Then, the solution at the emission side is

$$v(B) = \sum_{mn} \frac{f_{mn}}{Z^B_{mn}} \, \phi_{mn}(x, y)$$

and the radiated power can be evaluated using the Rayleigh integral

$$\Pi_{rad} = \mathbf{v}^\star_{mn} \, \mathbf{\Phi}^{\mathrm{T}} \, \mathbf{R} \, \mathbf{\Phi} \, \mathbf{v}_{mn}$$

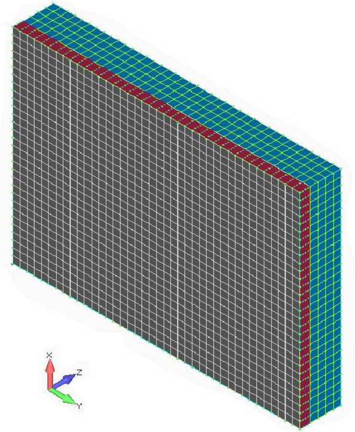
PROS

- The system is entirely solved by TMM
- the locally reacting behavior hypothesis has been removed
- fast

CONS

- infinite flat systems
- only the modal behavior of one subsystem can be considered
- only continuity of normal stress and displacement

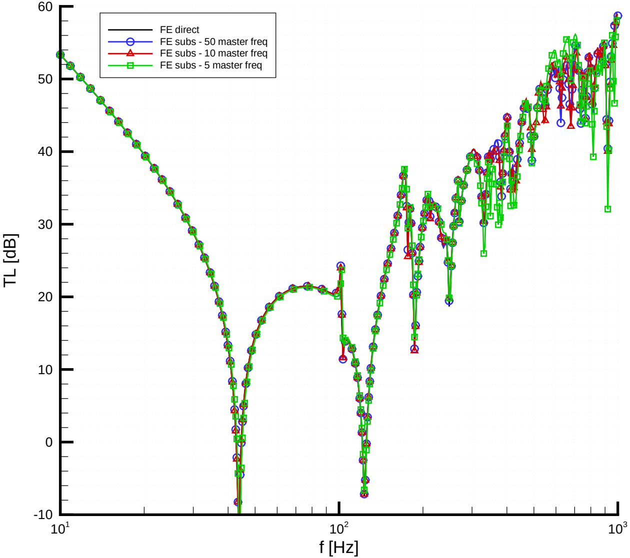
Benchmark description

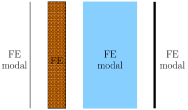


- $.5 \times .7 \times .1$ m^3
- 2 mm thick SS aluminium skin panel excited by 45°/45° plane wave
- 12 mm thick SS composite trim panel
- 25 mm thick melamine foam sliding onto the lateral faces
- 75 mm thick air gap
- band of interest 10-1000 Hz
- $30 \times 40 \times 8$ linear FEs
- 500 frequency points, constant log-step
- modes up to 3 KHz

Results (1)

Full FE substructuring - efficiency vs accuracy

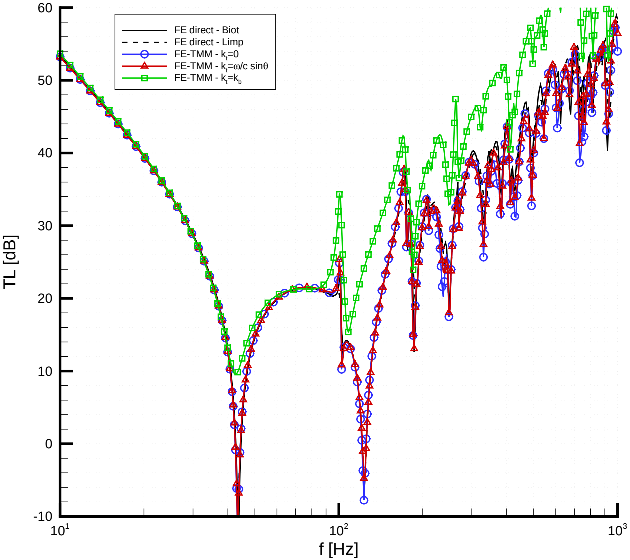


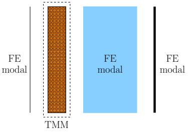


- number of master frequencies (trade-off)
- cpu time:
  direct 9h
  subs 10mn/master freq.

Results (2)

FE-TMM - model 1

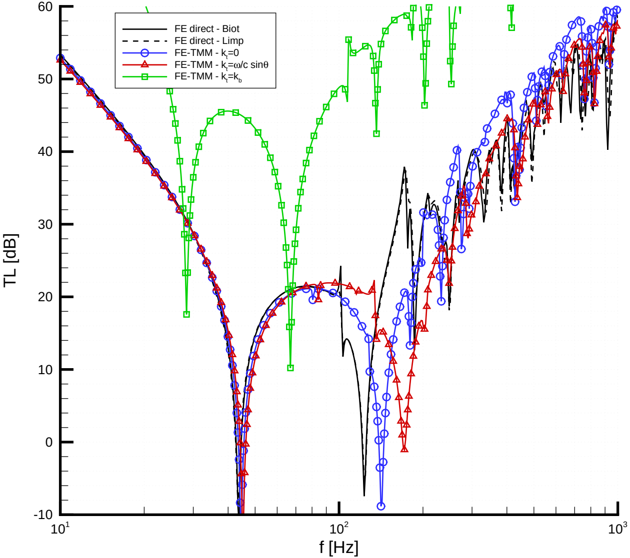


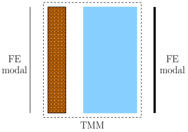


- $k_t = \frac{\omega}{c} \sin\theta$ gives almost perfect correlation
- cpu time:
  direct Biot 9h
  direct Limp 5.5h
  FE-TMM <10mn

Results (3)

FE-TMM - model 2

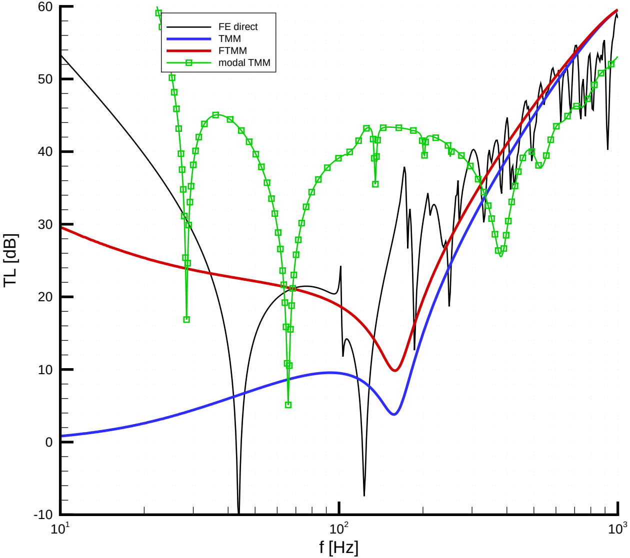


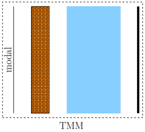


- $k_t = \frac{\omega}{c} \sin\theta$ gives the right tendency above 300 Hz
- below 300 Hz the cavity lateral modes cannot be neglected
- cpu time:
  direct Biot 9h
  direct Limp 5.5h
  FE-TMM <10mn

Results (4)

modal TMM

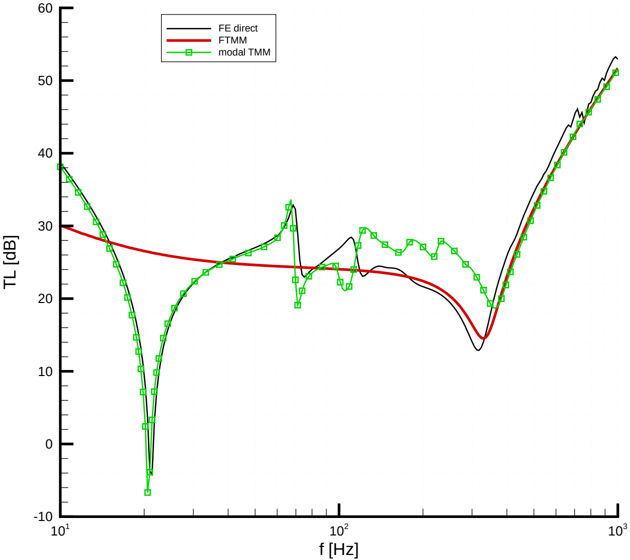


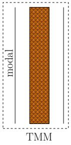


- the tendency is respected only at higher frequencies
- huge errors below 400 Hz

Results (5)

modal TMM - plate/foam/screen system





- good correlation is observed
- discrepancies around the coincidence freq.
- cpu time:
  direct 4h
  modal TMM 2mn

Conclusions

FINAL ASSESSMENT

- FE subs: the trade-off between accuracy and efficiency is the key
- FE-TMM: fast and quite accurate, k_t must be correctly selected
- modal TMM: very fast but limited to simple systems

ANSWER TO THE SCIENTIFIC QUESTION

- the FE modeling of the poroelastic material can be avoided
- the lateral modes of the cavity may be important (depending on the desired accuracy)
- the modes of the skin panel alone are not sufficient to estimate the TL of the entire DWL system.

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Perspectives

- further investigations and improvements of the modal TMM
- acoustic and mechanical excitations
- curvature effect
- extension to non-standard absorbent materials (i.e. passive/active inclusions)

Thanks for your attention!

Acknowledgements: NSERC, Bombardier, PWC, Bell Helicopter