Modeling of a perforated solid with dead-end porosity by the transfer matrix method

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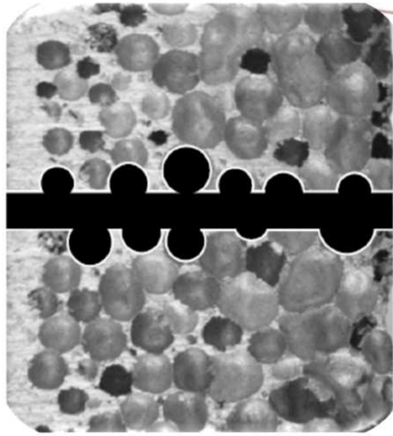
Introduction

Examples of **complex** media with dead-end porosities:

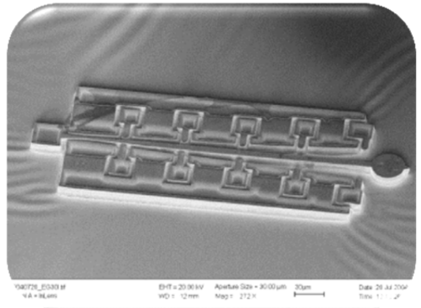
- Hydrogeology (e.g. soils);

- Perforated closed-cell metallic foams



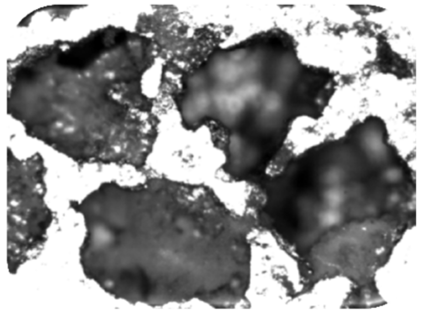
F. Chevillotte et al. JASA 2010

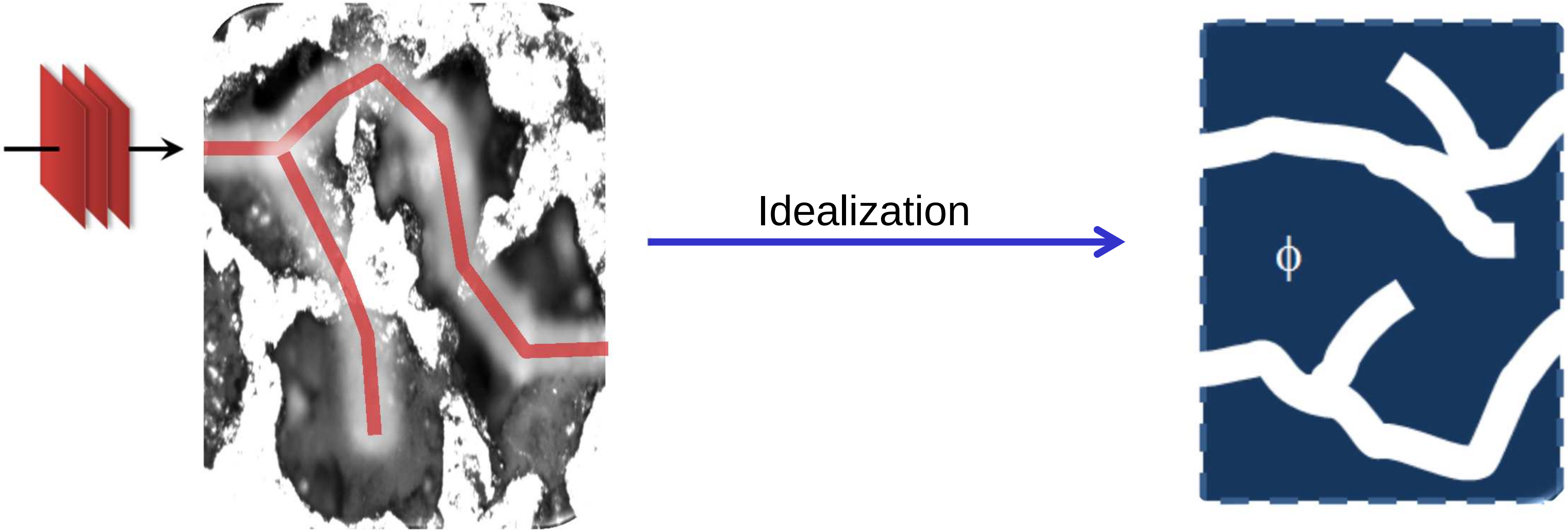
- Biophysical (Neurobiological application on brains);



- Meta-materials (man designed materials).

- Partially reticulated metallic foams





→ Partially reticulated porous medium contains interconnected pores

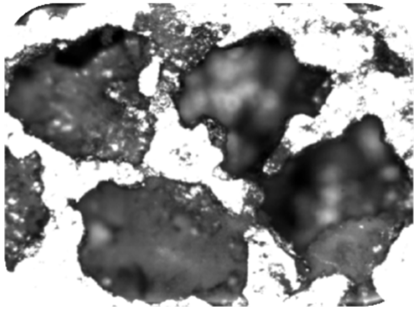
→ Some pores are fully opened to surrounding medium (= BIOT PORES)

→ Some pores are not opened to surrounding medium (= DEAD END PORES)

Objectives

GLOBAL

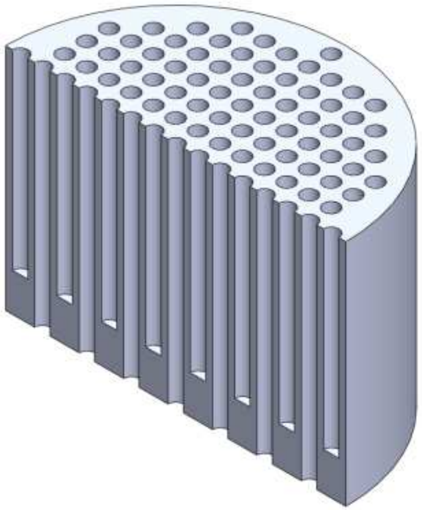
- Model the acoustic behavior of the partially reticulated metallic foams



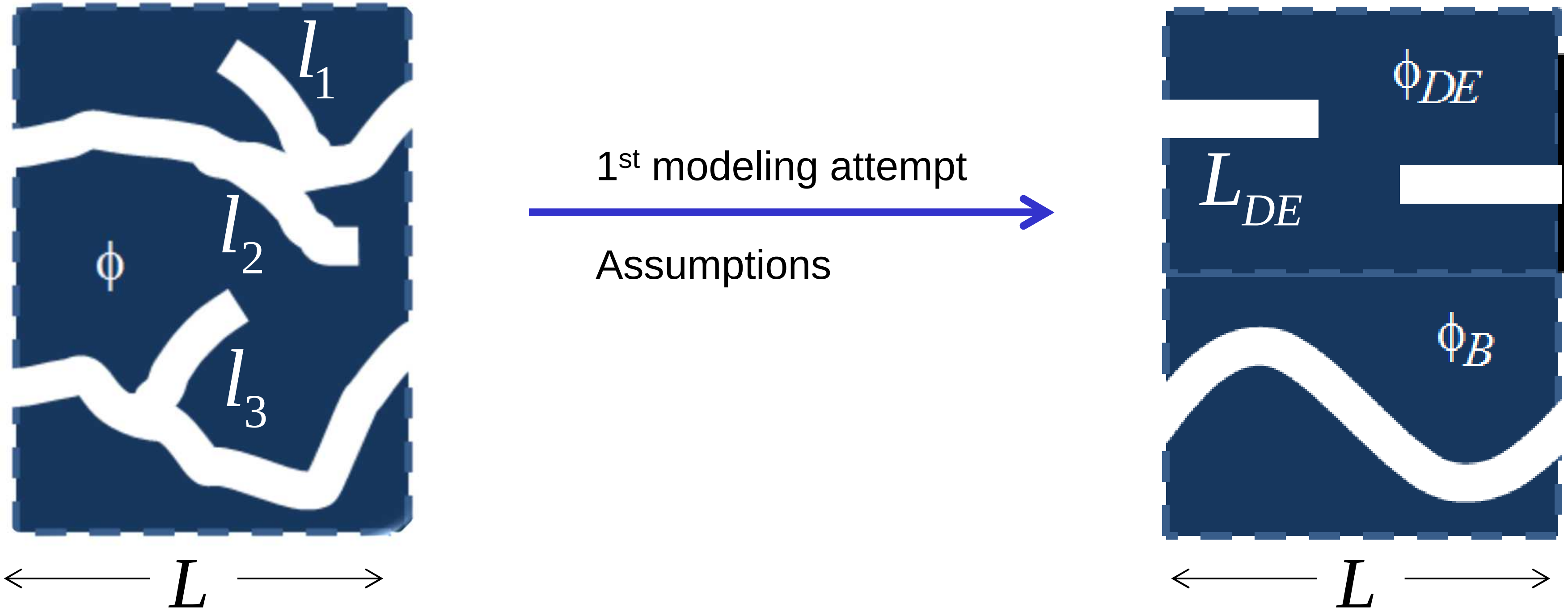
INITIAL (this presentation)

- Propose a first model for these foams using the transfer matrix approach

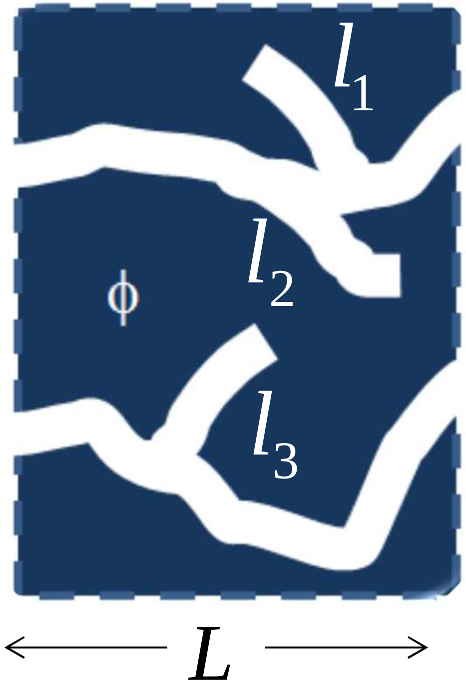
First model based on a perforated solid with dead-end porosity



Modeling

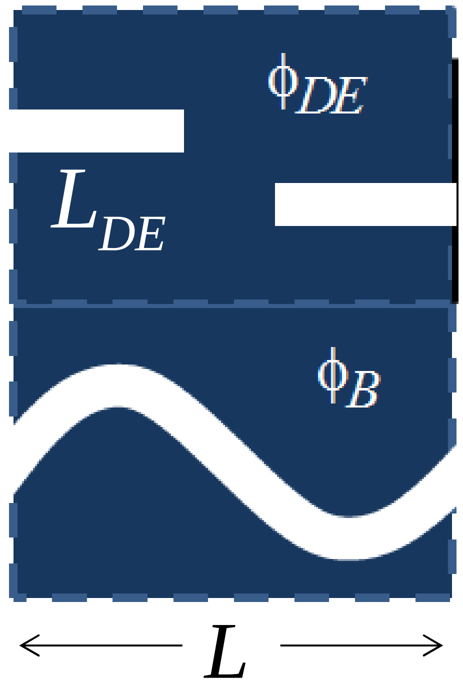


1. All dead-end (DE) pores are brought to the surfaces
2. Localized reaction is assumed
3. Medium is divided in two PARALLEL layers (in time and space):
   - One layer with pores fully opened to surrounding medium (ϕ_B , L)
   - One layer with DE pores (ϕ_{DE} , L_{DE})



1st modeling attempt

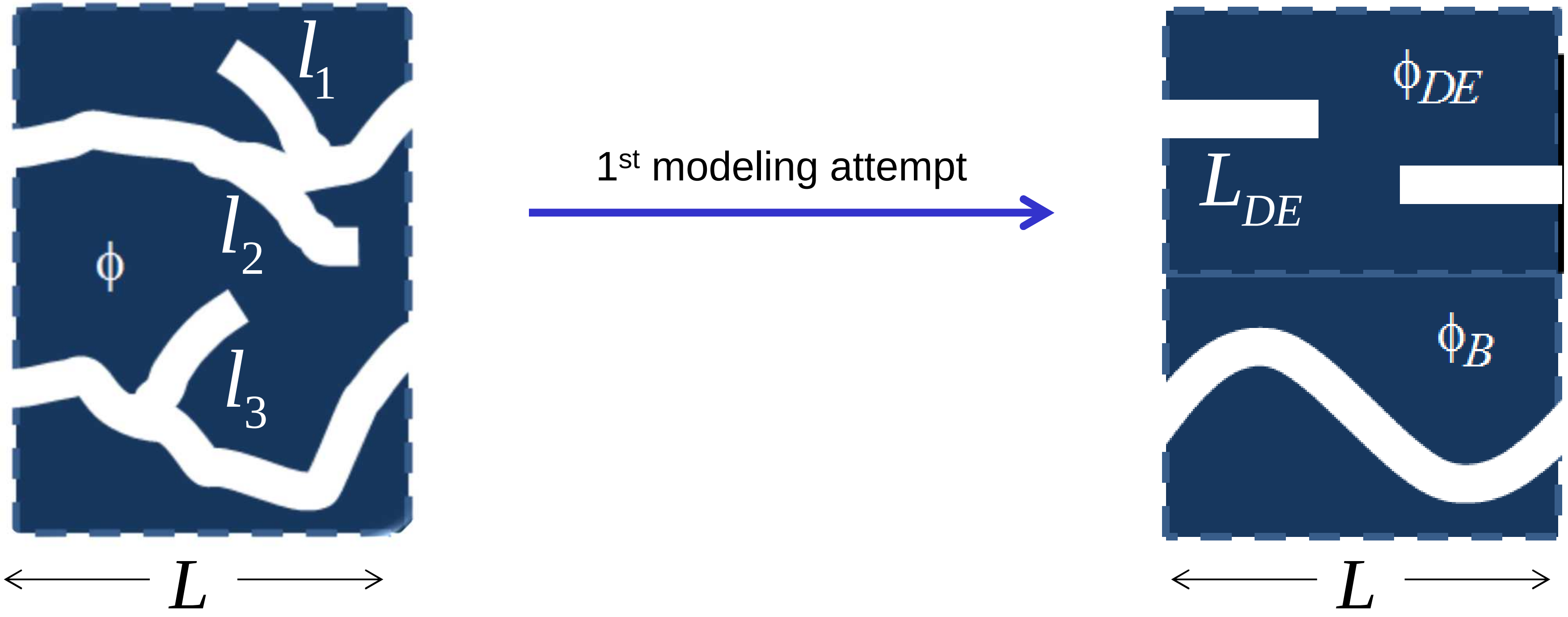
Assumptions



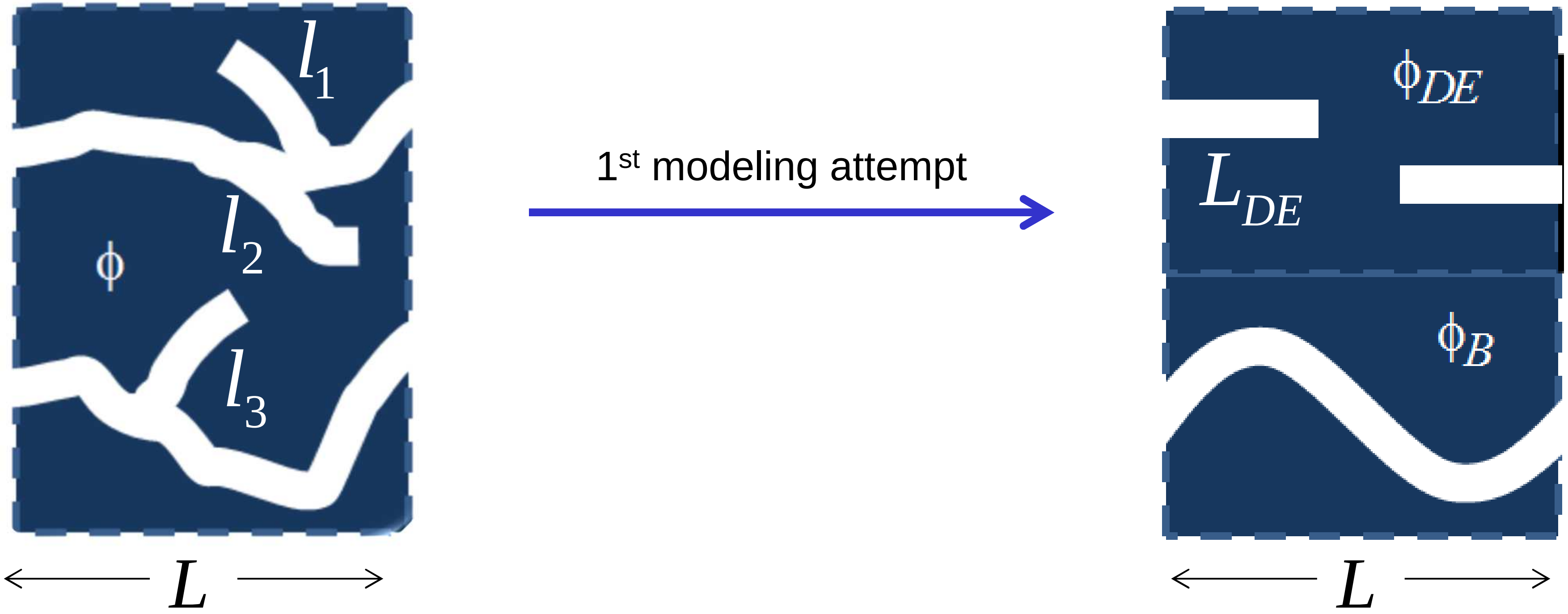
4. Acoustic Admittance is the sum of the acoustic admittances of the two parallel layers

$$Y(\phi) = Y_1(\phi_B) + Y_2(\phi_{DE})$$

$$\phi = \phi_B + \phi_{DE}$$



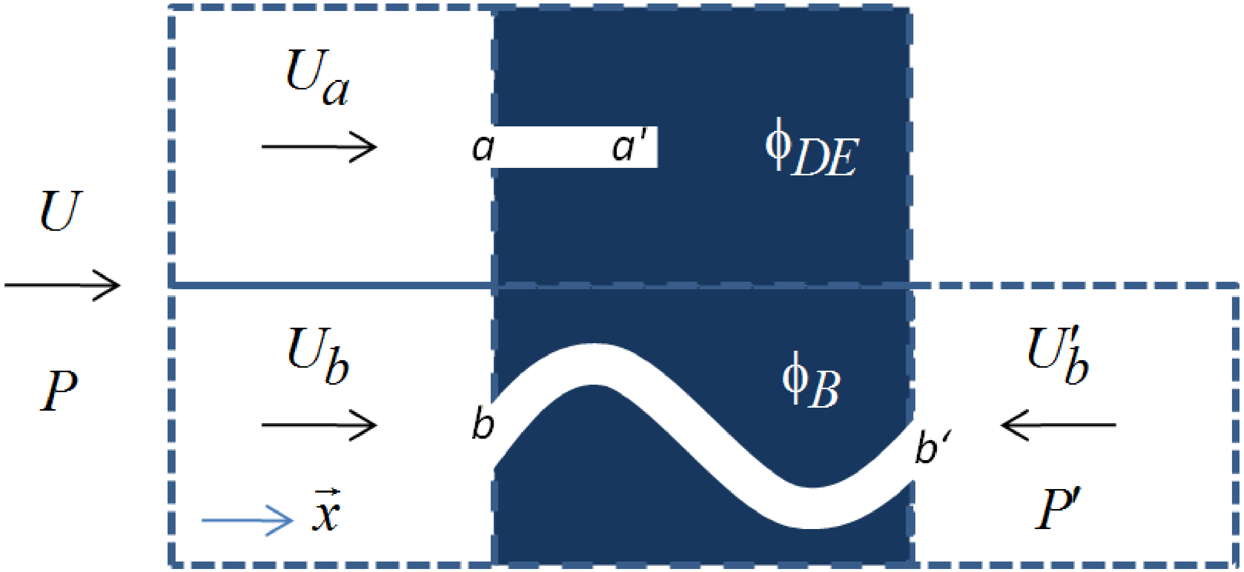
- Open pores are characterized by equivalent fluid (JCA model) macroscopic parameters
  - BIOT porosity (ϕ_B)
  - Static airflow resistivity (σ_B)
  - Tortuosity ($\alpha_{\infty B}$)
  - Characteristic viscous and thermal lengths (Λ_B, Λ'_B)



- ❑ Dead-End pores are characterized by equivalent fluid (JCA model) macroscopic parameters
  - o σ_B , $\alpha_{\infty B}$, Λ_B, Λ'_B
  - o DE porosity (ϕ_{DE})
  - o DE length (L_{DE})

$$L_{DE} = \iiint_{V_{DE}} l\,dV \Big/ \iiint_{V} dV$$

Transfer Matrix – Non symmetric configuration



RECALL: ASSUMPTION 4

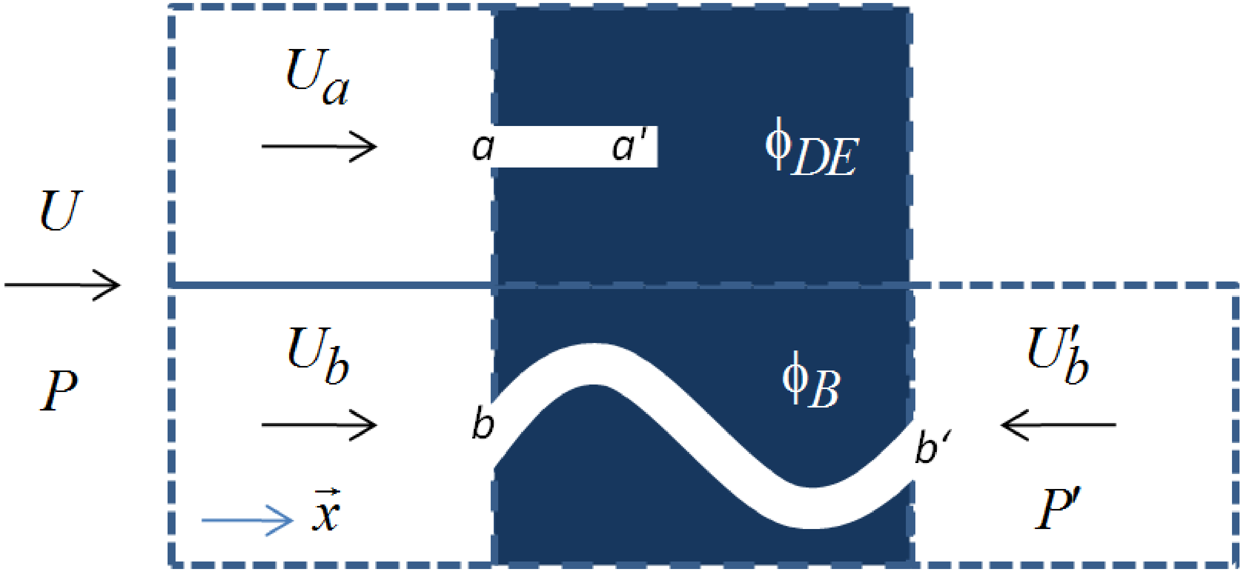
$$Y(\phi) = Y_1(\phi_B) + Y_2(\phi_{DE})$$

DEAD END LAYER

$$[T]^{DE} = \begin{bmatrix} t_{11}^{de} & t_{12}^{de} \\ t_{21}^{de} & t_{22}^{de} \end{bmatrix} = \begin{bmatrix} \cos(\bar{k}_{DE} l_{DE}) & j\bar{Z}_{DE} \sin(\bar{k}_{DE} l_{DE}) \\ \dfrac{j}{\bar{Z}_{DE}} \sin(\bar{k}_{DE} l_{DE}) & \cos(\bar{k}_{DE} l_{DE}) \end{bmatrix}$$

$$\begin{Bmatrix} P_a \\ U_a \end{Bmatrix} = [T]^{DE} \begin{Bmatrix} P'_a \\ -U'_a \end{Bmatrix} \rightarrow \boxed{\begin{Bmatrix} U_a \\ U'_a \end{Bmatrix} = [Y]^{DE} \begin{Bmatrix} P_a \\ P'_a \end{Bmatrix}}$$

Transfer Matrix – Non symmetric configuration



RECALL: ASSUMPTION 4

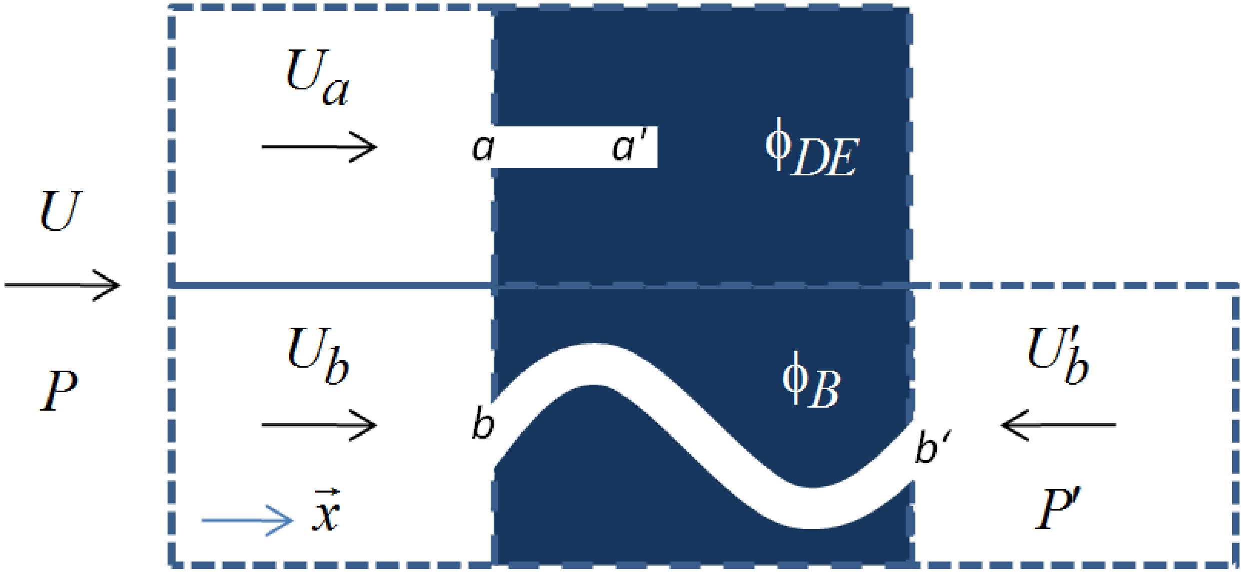
$$Y(\phi) = Y_1(\phi_B) + Y_2(\phi_{DE})$$

BIOT LAYER

$$[T]^B = \begin{bmatrix} t_{11}^b & t_{12}^b \\ t_{21}^b & t_{22}^b \end{bmatrix} = \begin{bmatrix} \cos(\bar{k}_B l) & j\bar{Z}_B \sin(\bar{k}_B l) \\ \frac{j}{\bar{Z}_B} \sin(\bar{k}_B l) & \cos(\bar{k}_B l) \end{bmatrix}$$

$$\begin{Bmatrix} P_b \\ U_b \end{Bmatrix} = [T]^B \begin{Bmatrix} P'_b \\ -U'_b \end{Bmatrix} \rightarrow \boxed{\begin{Bmatrix} U_b \\ U'_b \end{Bmatrix} = [Y]^B \begin{Bmatrix} P_b \\ P'_b \end{Bmatrix}}$$

Transfer Matrix – Non symmetric configuration



RECALL: ASSUMPTION 4

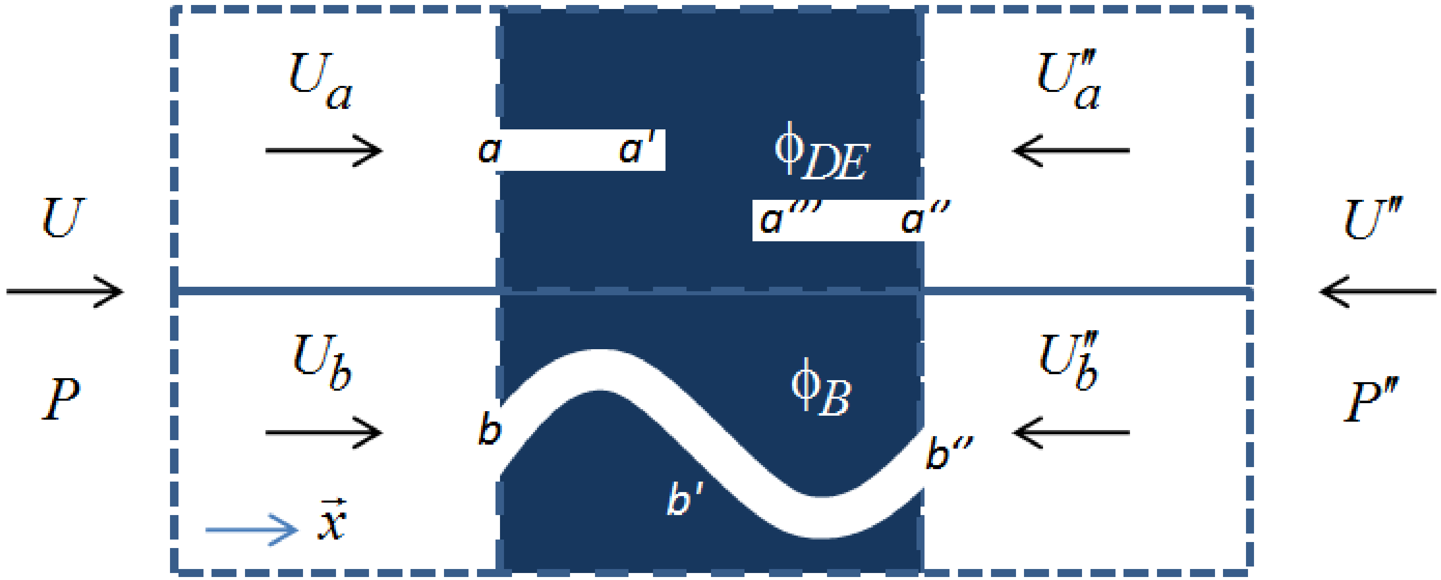
$$Y(\phi) = Y_1(\phi_B) + Y_2(\phi_{DE})$$

IMPOSING CONTINUITY CONDITIONS (Pressure and Flow rate)

$$\begin{Bmatrix} P \\ U \end{Bmatrix} = [T]^{NS} \begin{Bmatrix} P_b' \\ -U_b' \end{Bmatrix}$$

$$[T]^{NS} = \frac{1}{y_{12}^b} \begin{bmatrix} -y_{22}^b & -1 \\ \left(y_{12}^b\right)^2 - y_{22}^b \left(y_{11}^b + y_{11}^{de} - \frac{\left(y_{12}^{de}\right)^2}{y_{22}^{de}} \right) & -\left(y_{11}^b + y_{11}^{de} - \frac{\left(y_{12}^{de}\right)^2}{y_{22}^{de}} \right) \end{bmatrix}$$

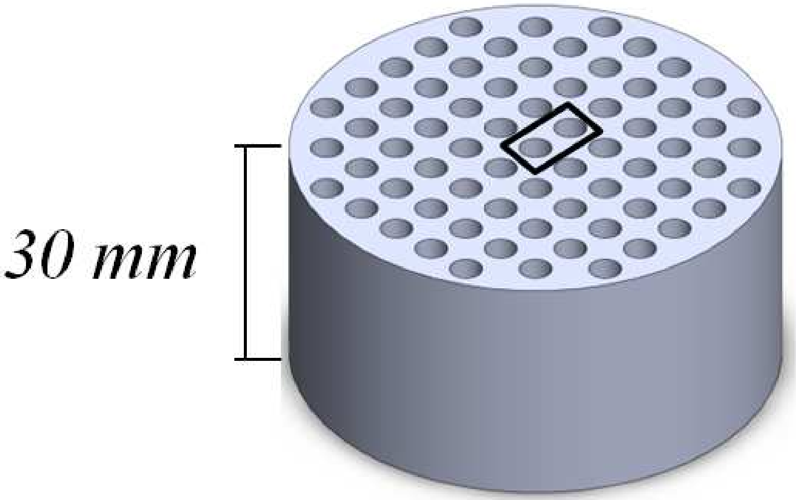
Transfer Matrix – Symmetric configuration



SIMILAR APPROACH FOR SYMMETRIC CONFIGURATION

$$\begin{Bmatrix} P \\ U \end{Bmatrix} = [T]^S \begin{Bmatrix} P'' \\ -U'' \end{Bmatrix}$$

Validation 1 : Perforated Solid with DE



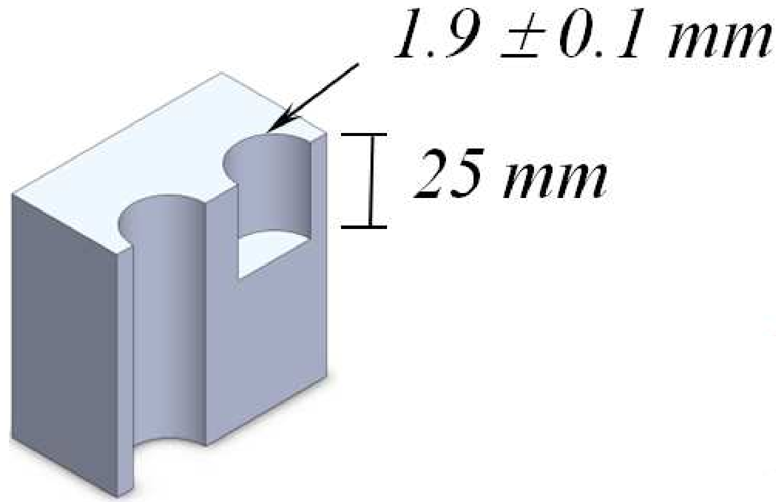
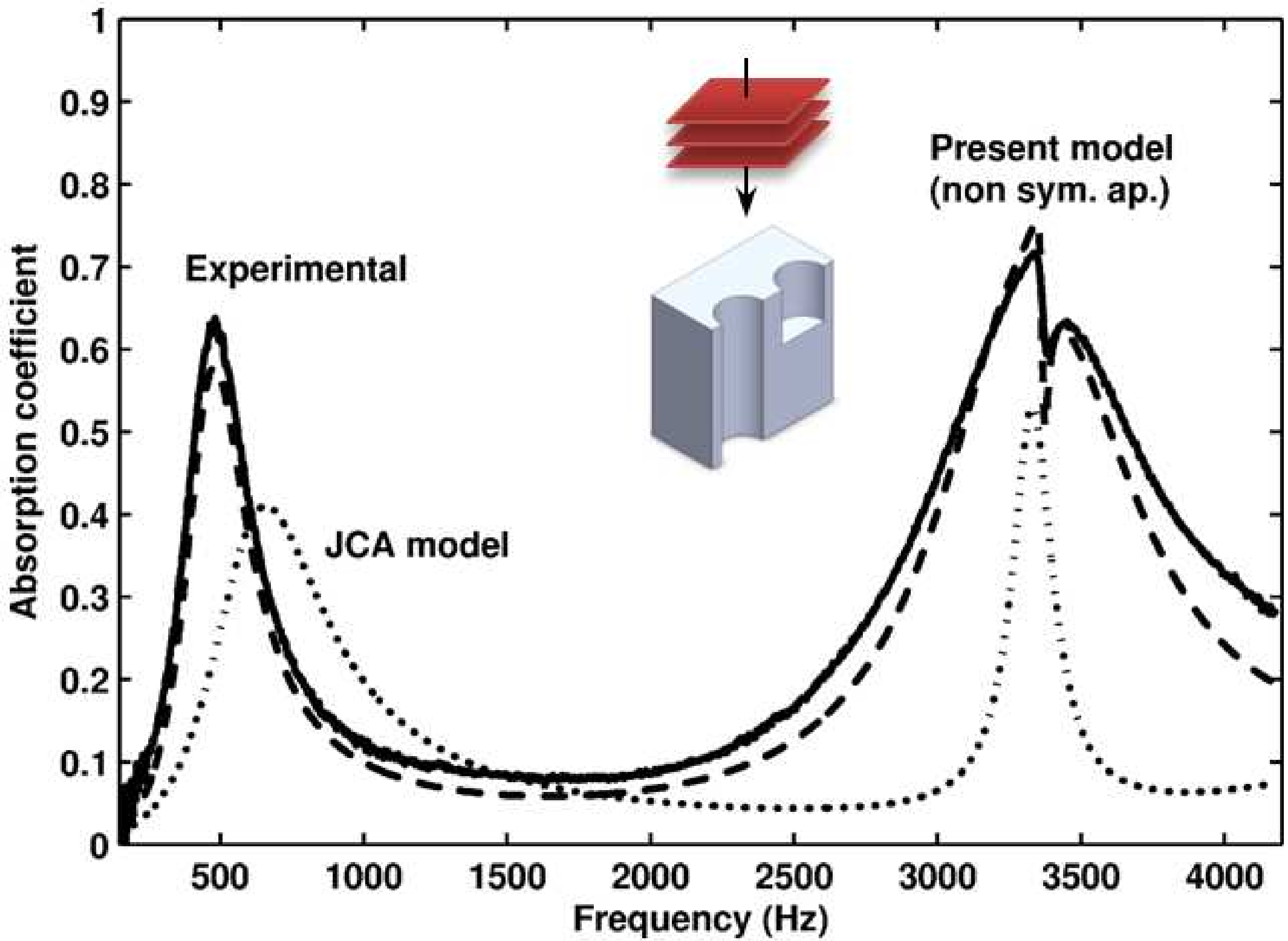


TABLE I. Dead-end parameters of the non-symmetric simplified sample.

Total porosity (%)	ϕ_B (%)	ϕ_{DE} (%)	l_{DE} (mm)
27.5	14	13.5	25

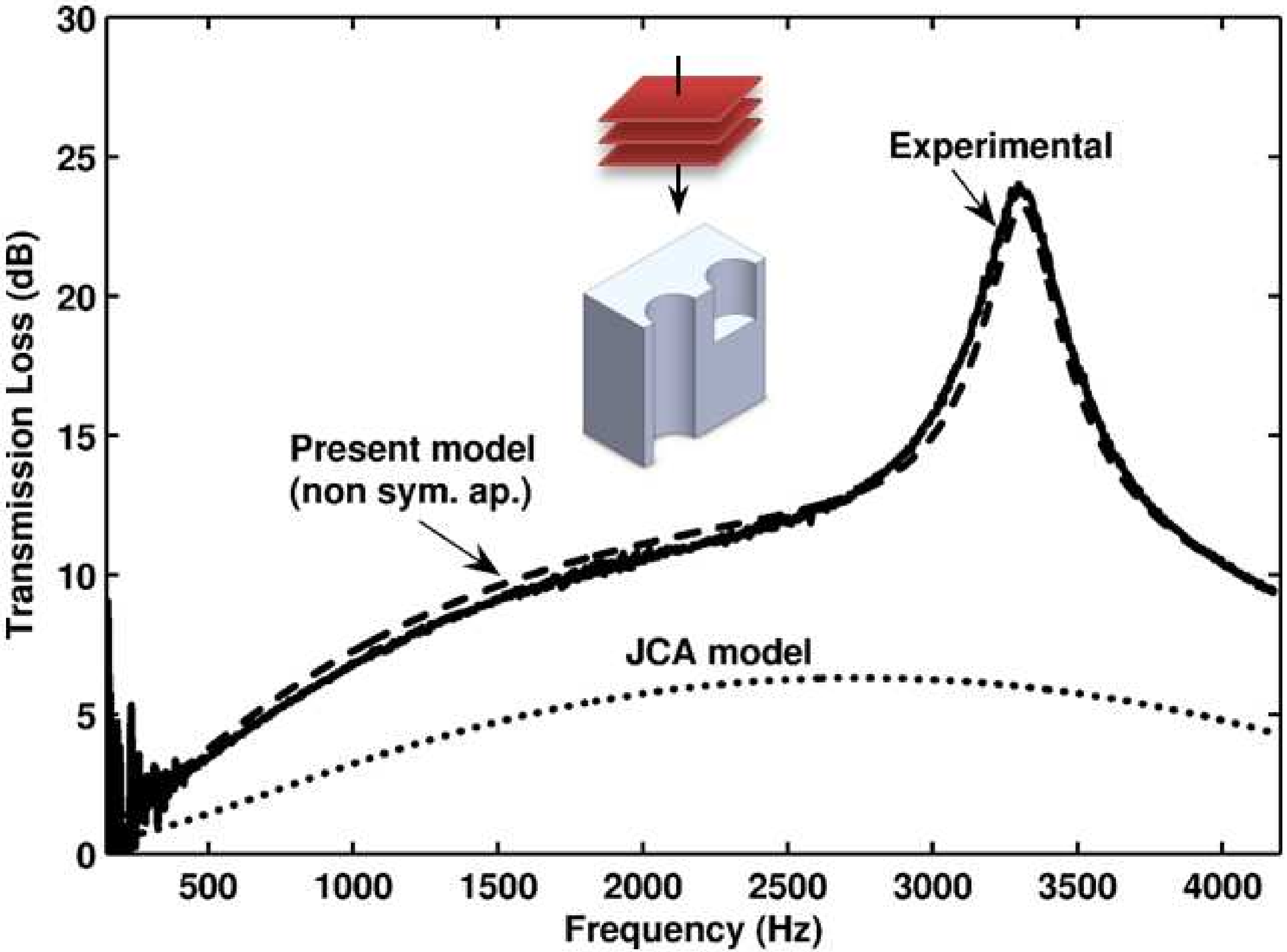
$\Lambda' = d / 2$ $\qquad \Lambda = d_{min} / 2$ $\qquad \alpha_\infty = 1$ $\qquad \sigma = 32\eta/\phi\, d^2$

Validation 1 : Perforated Solid with DE



Sample is backed by 50-mm thick air cavity

Validation 1 : Perforated Solid with DE



Validation 2 : Aluminum Foam with DE

TABLE II. Johnson-Champoux-Allard (JCA) parameters of the aluminum foam sample.

JCA parameters	Λ (μm)	Λ' (μm)	α_∞	σ (Pa s/m^2)	ϕ (%)
Mean value	101	352	2.25	19 713	64.5
Standard deviation	4	14	0.05	300	3

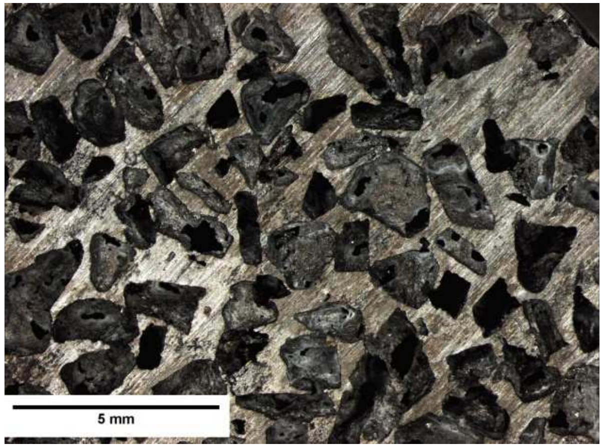
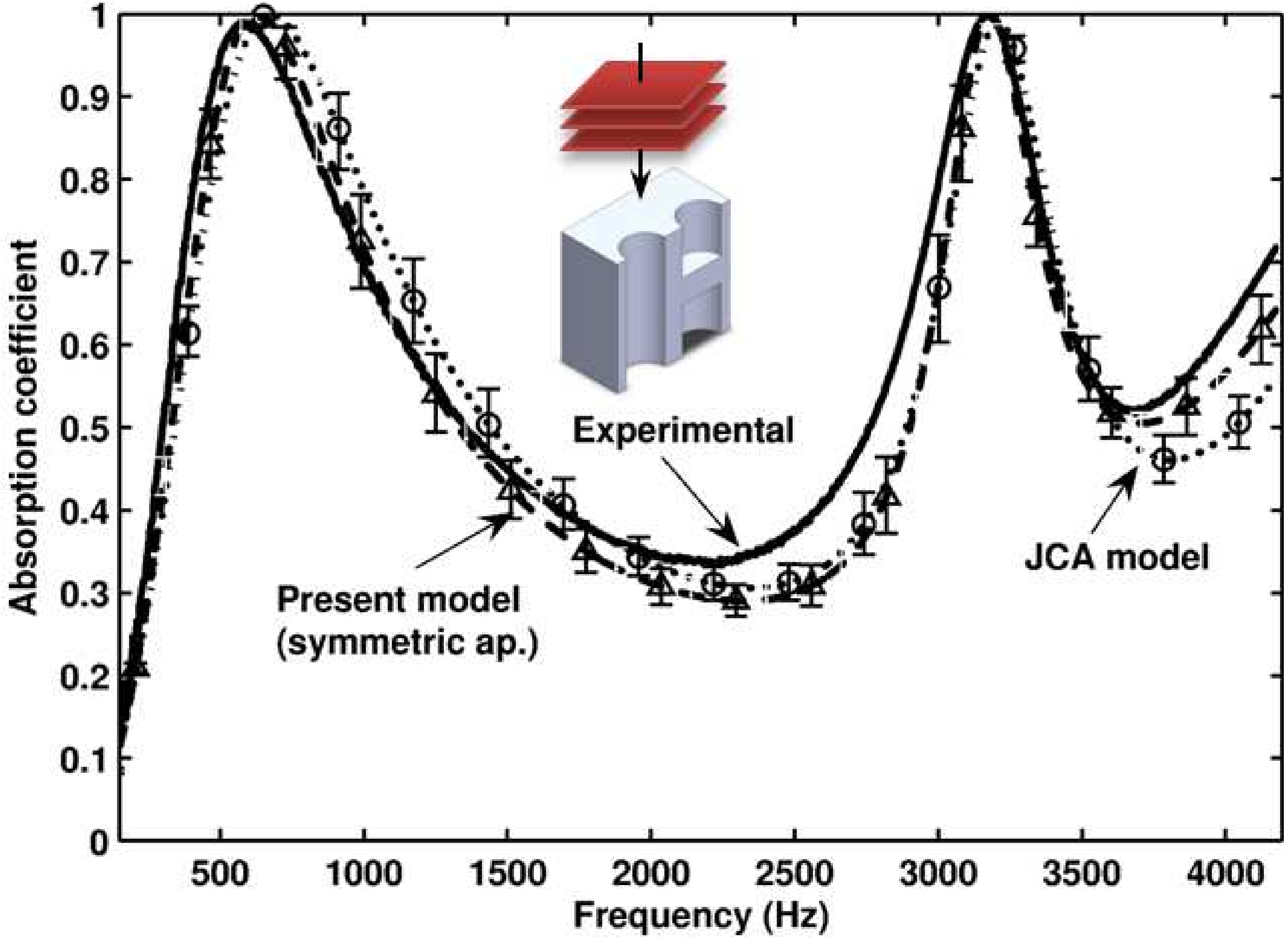


TABLE III. Dead-end parameters of the aluminum foam sample (fitting and experimental approaches).

Total porosity (%)	ϕ_B (%)	ϕ_{DE} (%)	l_{DE} (mm)
64.5 ± 3	≈ 55	≈ 7.5	$\approx 7d_{cell}$

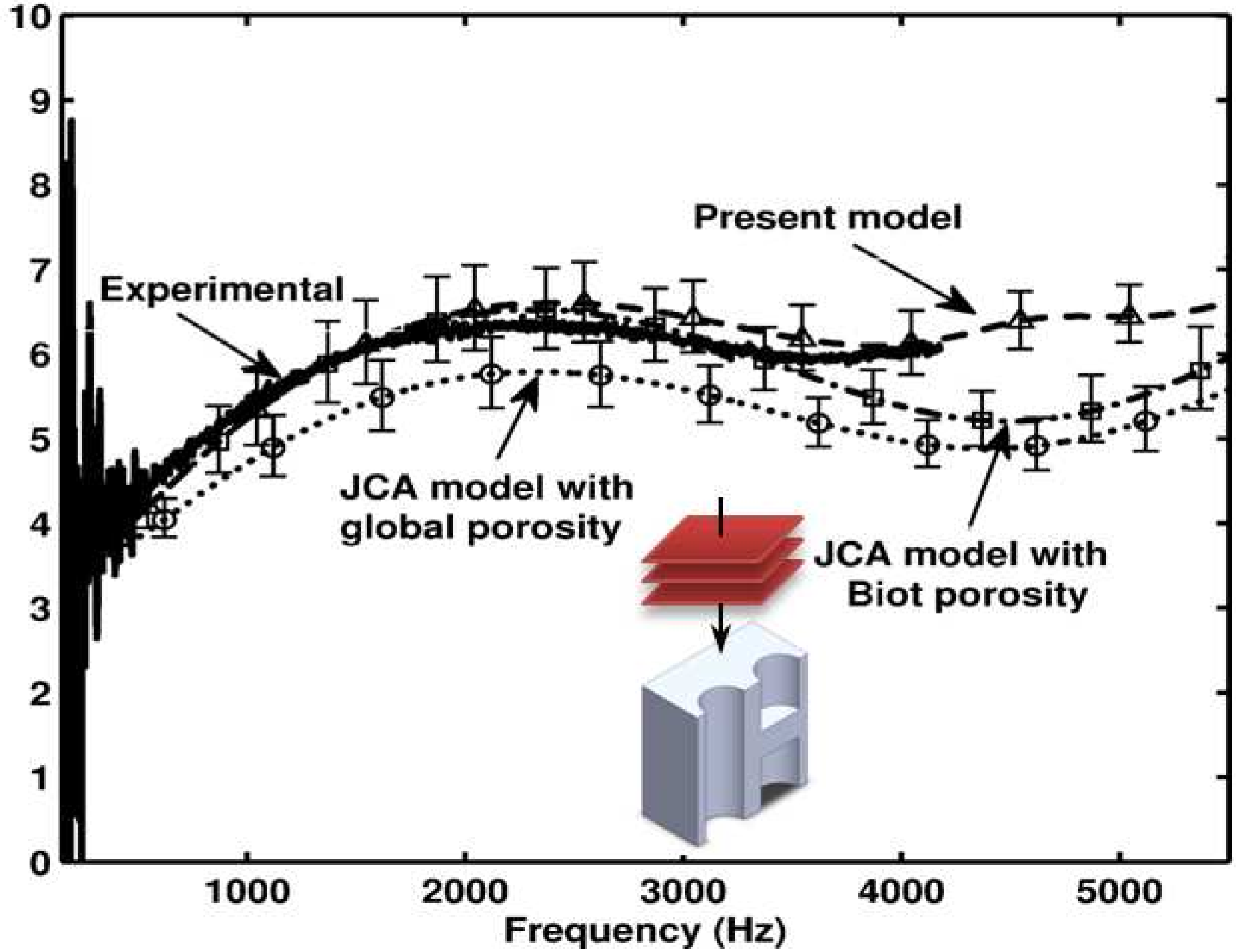
$$l_{DE} = n\, d_{cell}$$

Validation 2 : Aluminum Foam with DE



Sample is backed by 50-mm thick air cavity

Validation 2 : Aluminum Foam with DE



Conclusion

Perforated solid with dead-end porosity

- JCA model cannot reproduce measurements
- Proposed TM model can reproduce measurements

Aluminium foam with dead-end porosity

- JCA model yields good predictions of absorption
- JCA model lacks accuracy in transmission at high frequencies
- Proposed TM model seems to reproduce the main features of the measurements

Perspective

Test the model on other partially reticulated foams

Perform simulations at the local scale to better understand the effects of dead-end pores with respect with their shape and size

Establish methods for characterizing the macroscopic properties of the Dead-End porosities