Absorption of rigid frame porous grating with various shape rigid or soft inclusions.

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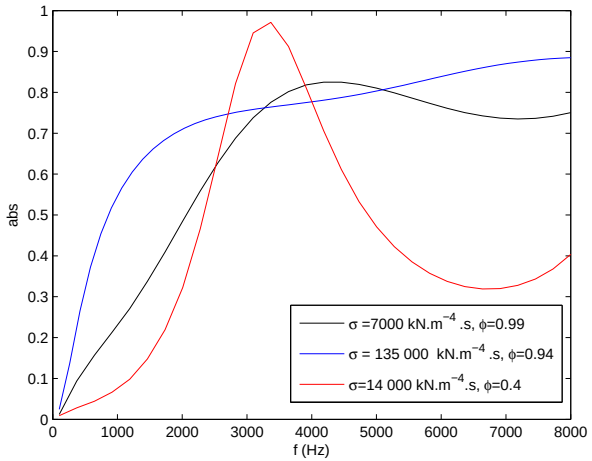
Introduction

- Objectives
- State of the art

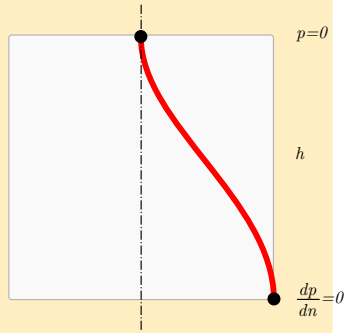
Objectives



Absorption of various homogeneous porous material (2cm height)

Phenomenons involved in absorption in a porous slab

- Viscous loss ✓
- Thermal loss ✓
- Quarter-wave resonance ✓
- Trapped mode ✗



Heterogeneity in the porous may increase the entrapment of the energy

Why not add periodical inclusions ?

- Air inclusions (ie double porosity materials) : Atalla et al., 2001 ; X. Olny et al. 2005, F. Sgard et al. 2011.
- Rigid inclusions : J.-P. Groby et al., 2011.
- Porous inclusions : X. Bécot et al., 2010.

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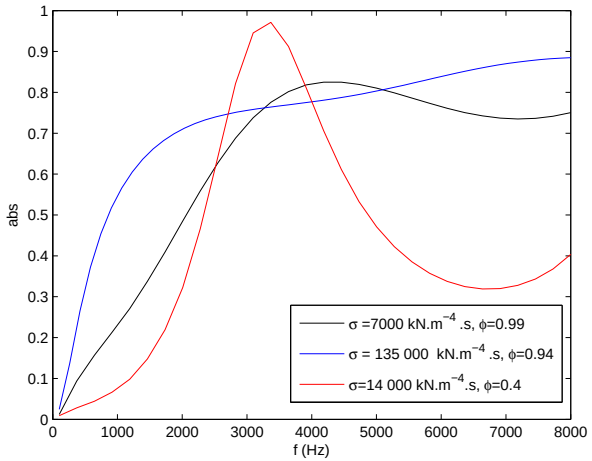

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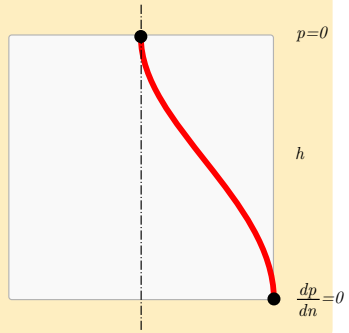

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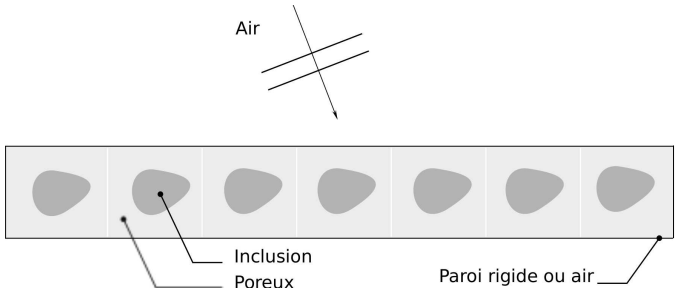
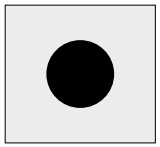
State of the art

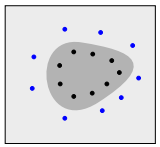
Interests

- Tune the apparent properties of the porous material, e. g.
  - Small flow resistivity ▶ Rigid inclusions
  - High ▶ Air inclusions (double porosity)
  - Between ▶ Porous material inclusions
- Add resonance below quarter-wave mode
- Build structuring acoustic absorbing packages

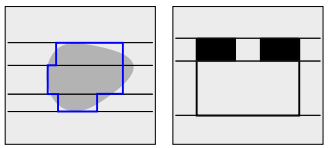




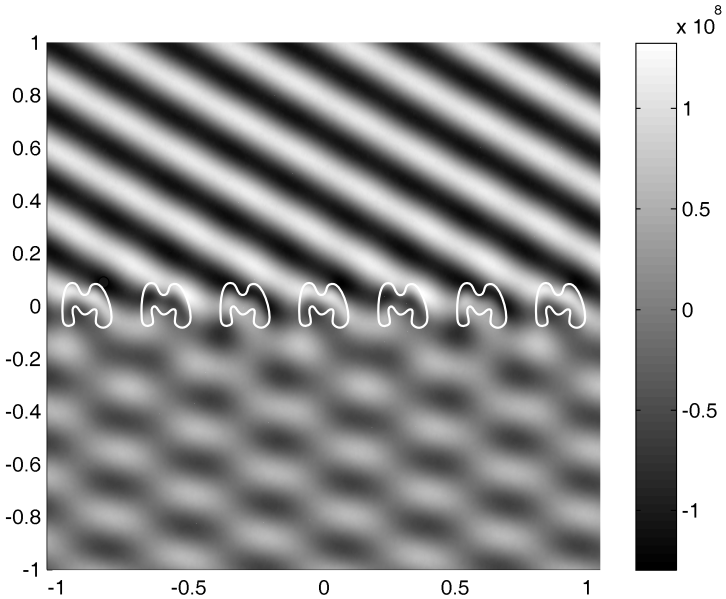
Multipole method
(Groby et al., 2008)



Method of fundamental solution
(Nennig et al., 2011)



Layered mode matching
(Allard et al., 2011)

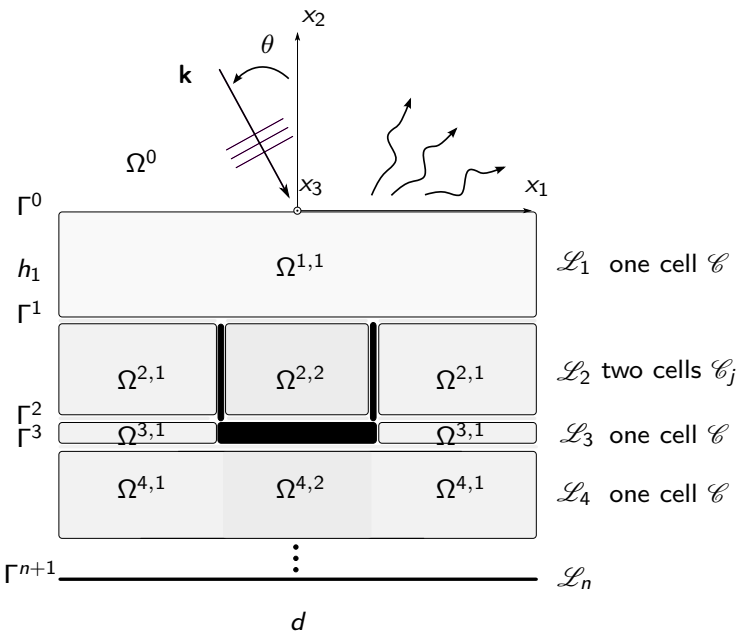


Scattering by a poroelastic array of 'M'

Mode matching method

- Overview
- Modal decomposition in each layer
- Matching schemes

Overview (1)



Consider the computation of the acoustic absorption of a multi-layered d-periodic porous material impinged by a plane wave

$$p^{\mathrm{inc}} = A_0 \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}}. \tag{1}$$

Each of the $N_{\mathscr{L}}$ layers may include

- another material $\Omega^{i,j}$ (see $\mathscr{L}_4$) ;
- rigid scatterers (see $\mathscr{L}_2$ and $\mathscr{L}_3$)

in cells $\mathscr{C}_j$.

At the interfaces Γ^i ($i = 0,\ N_{\mathscr{L}} + 1$), continuity conditions are applied

As the problem is periodic and the excitation is due to a plane wave, each field (X) satisfies the Floquet-Bloch relation

$$X(x_1 + d, x_2) = X(x_1, x_2)\vartheta, \tag{2}$$

with $\vartheta = \mathrm{e}^{\mathrm{i}k_1 d}$.

Overview (2)

In each domain $\Omega^{i,j}$ and in the surrounding air domain Ω^0, the pressure fulfills the Helmholtz's equation ($\mathrm{e}^{-\mathrm{i}\omega t}$)

$$\Delta p(\mathbf{x})^{i,j} + (k^{i,j})^2 p(\mathbf{x}) = 0, \tag{3}$$

Problem variables are separable, hence

$$p(\mathbf{x})^{i,j} = \chi(x_1)^{i,j} \mathcal{Y}(x_2)^{i,j}. \tag{4}$$

Above the material, previous relation, the radiating condition and the Floquet's relation lead to

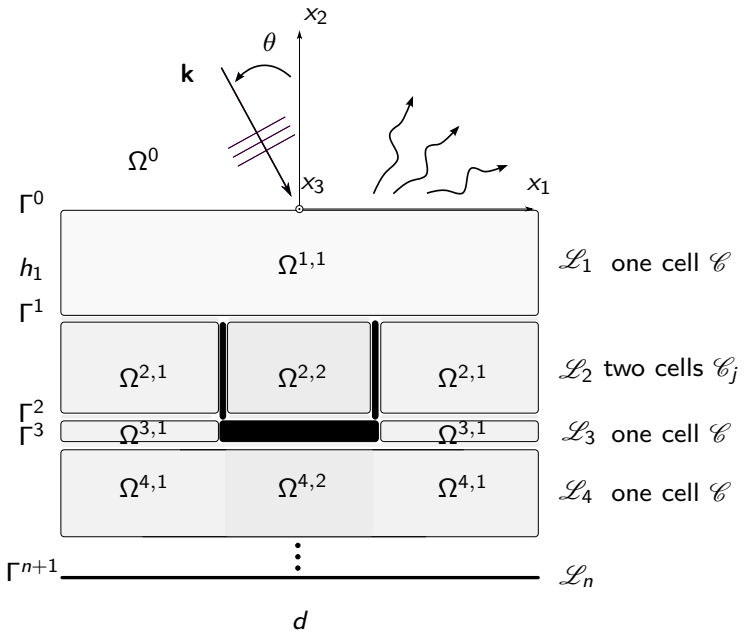
$$p(\mathbf{x})^0 = \sum_{n\in\mathbb{Z}} A_n^0 \frac{1}{\sqrt{d}} \mathrm{e}^{\mathrm{i}\alpha_n^0 x_1} \mathrm{e}^{\mathrm{i}\beta_n^0 x_2}, \forall \mathbf{x} \in \Omega^0, \tag{5}$$

with the wave numbers $\alpha_n^0 = k_1 + n\frac{2\pi}{d}$ and $\beta_n^0 = \sqrt{(k^0)^2 - \alpha_n^2}$. The pressure field restriction into the cell $\mathscr{C}^{i,j}$ is sought using the modal expansion

$$p(\mathbf{x})^{i,j} = \sum_{n\in\mathbb{N}} \left(A_n^{i,j} \chi_n^{i,j}(x_1) \mathrm{e}^{\mathrm{i}\beta_n^{i,j} x_2} + B_n^{i,j} \chi_n^{i,j}(x_1) \mathrm{e}^{-\mathrm{i}\beta_n^{i,j} x_2} \right), \forall \mathbf{x} \in \mathscr{C}^{i,j}. \tag{6}$$

Here $A_n^{i,j}$ and $B_n^{i,j}$ are the traveling wave amplitudes, $\chi_n^{i,j}$ et $\beta_n^{i,j}$ are the eigenfunctions and the eigenvalues (vertical), respectively. ▶ **Have to be found !**

Modal decomposition in each layer



Boundary conditions

For each cell $\mathscr{C}^{i,j}$, the pressure and the normal velocity continuity are applied between material $\Omega^{i,j}$.
Normal velocity vanish on rigid wall
At $x_1 = d$, periodic boundary condition are applied

$$\chi(x_1 + d) = \chi(x_1)\vartheta, \tag{7}$$
$$\partial_{x_1}\chi(x_1 + d) = \partial_{x_1}\chi(x_1)\vartheta, \tag{8}$$

→ *We get a linear system involving the* $\beta_n^{i,j}$

Layer type

- Homogeneous ✓closed form
- Vert. rigid bar ✓closed form
- $\neq$ materials ✗closed form

Roots finding algorithm

The dispersion relation is solved numerically by mixing both algorithms

- *Winding number* method (Nennig, 2010).
- Newton-Raphson

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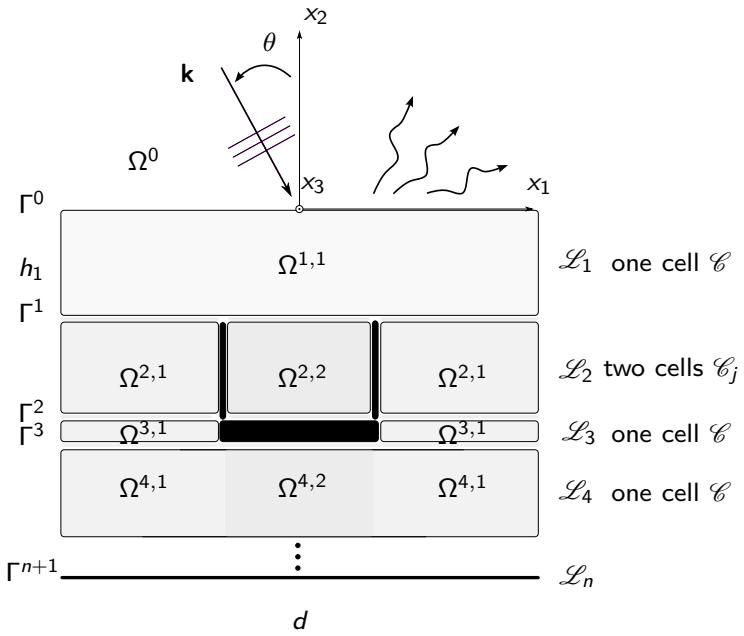

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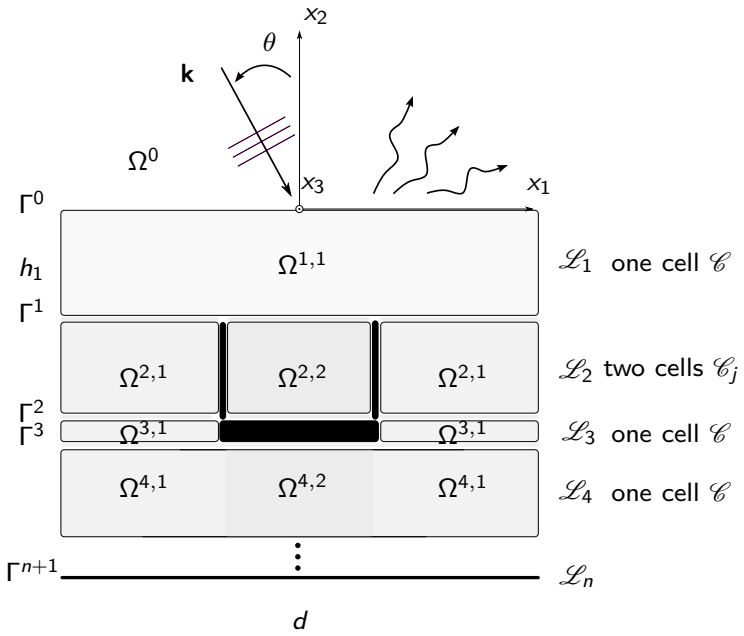

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Matching schemes on Γ_i (ie $x_2 = h_i$)

The pressure in each cell reads $p(\mathbf{x})^{i,j} = \sum_{n\in\mathbb{N}} \left(A_n^{i,j} \chi_n^{i,j}(x_1) \mathrm{e}^{\mathrm{i}\beta_n^{i,j} x_2} + B_n^{i,j} \chi_n^{i,j}(x_1) \mathrm{e}^{-\mathrm{i}\beta_n^{i,j} x_2} \right)$,

We put $p(\mathbf{x})^i = \bigcup_{j=1}^{N_{\mathscr{C}}^i} \left(p(\mathbf{x}_j)^{i,j}, \forall \mathbf{x}_j \in \mathscr{C}^i \right)$, the pressure p^i in the layer $\mathscr{L}_i$ with

Without singularity

Standard approach. Without rigid inclusion. The conditions must be satisfied

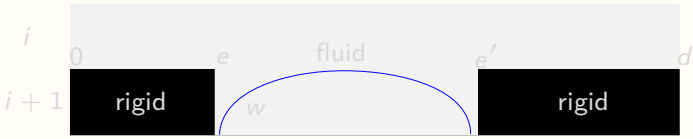
$$\int_0^d \chi_{m,p}^{i*} \left(p^i - p^{i+1} \right) \mathrm{d}x_1 = 0,$$

$$\int_0^d \chi_{m,v}^{i+1*} \left({v_2}^i - {v_2}^{i+1} \right) \mathrm{d}x_1 = 0.$$

- Orthogonality in some case (Allard, 2011)
- Gauss-Legendre quadrature rules are used

→ *Pb. with the discontinuities.*

With singularity at corners, (Homentcovschi, 2010).



On the interface ie $x_1 \in [e,\ e']$, the auxiliary **singular** velocity field is introduced $\tilde{v}_2 = w(x_1) \sum_k^\infty V_k C_k^\nu(x_1)$, with the weighting function

$$w(x_1) = \left[(e - x_1)(e' + x_1) \right]^{\nu - 1/2}$$

the coefficient V_k are new unknown and C_k^ν stand for the Gegenbauer's polynomials. The following conditions must be satisfied

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→ *A loop on the interfaces Γ^i yields the system* $\mathbf{KA} = \mathbf{F}$ *(size :* $(2 \cdot N \cdot N_{\mathscr{L}} + N)^2$*, with N the mode*

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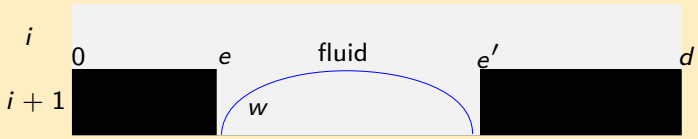

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Matching schemes > Resolution

The resolution is performed iteratively with a loop on the layers (Cummings, 1983). *The idea is to treat each interface separately by considering all incoming waves as known quantities and to use scattering matrix formalism...*

Algorithm

1. Initialization (previous freq. calculation, $\mathbf{0}$ ...)
2. For each interface $i = 0, ..., (N_{\mathscr{L}})$, solve

$$\mathbf{X}^i \mathbf{E}_X \begin{pmatrix} \mathbf{A}^i \\ \mathbf{B}^{i+1} \end{pmatrix} = \mathbf{Y}^i \mathbf{E}_Y \begin{pmatrix} \mathbf{A}^{i+1} \\ \mathbf{B}^i \end{pmatrix}, \tag{9}$$

with the diagonal matrices $\mathbf{E}_X$ et $\mathbf{E}_Y$ containing the phase factors for each depth h_i

$$\mathbf{E}_X = \mathrm{diag}\left(\mathrm{e}^{\mathrm{i}\beta_1^{i,+} h_i} \dots \mathrm{e}^{\mathrm{i}\beta_N^{i,+} h_i}, \mathrm{e}^{\mathrm{i}\beta_1^{i+1,-} h_i} \dots \mathrm{e}^{\mathrm{i}\beta_N^{i+1,-} h_i} \right),$$

$$\mathbf{E}_Y = \mathrm{diag}\left(\mathrm{e}^{\mathrm{i}\beta_1^{i+1,+} h_i} \dots \mathrm{e}^{\mathrm{i}\beta_N^{i+1,+} h_i}, \mathrm{e}^{\mathrm{i}\beta_1^{i,-} h_i} \dots \mathrm{e}^{\mathrm{i}\beta_N^{i,-} h_i} \right).$$

and with $\mathbf{X}^i$, $\mathbf{Y}^i$ the overlap integrals matrices (ie part of $\mathbf{K}$)

3. Goto 2

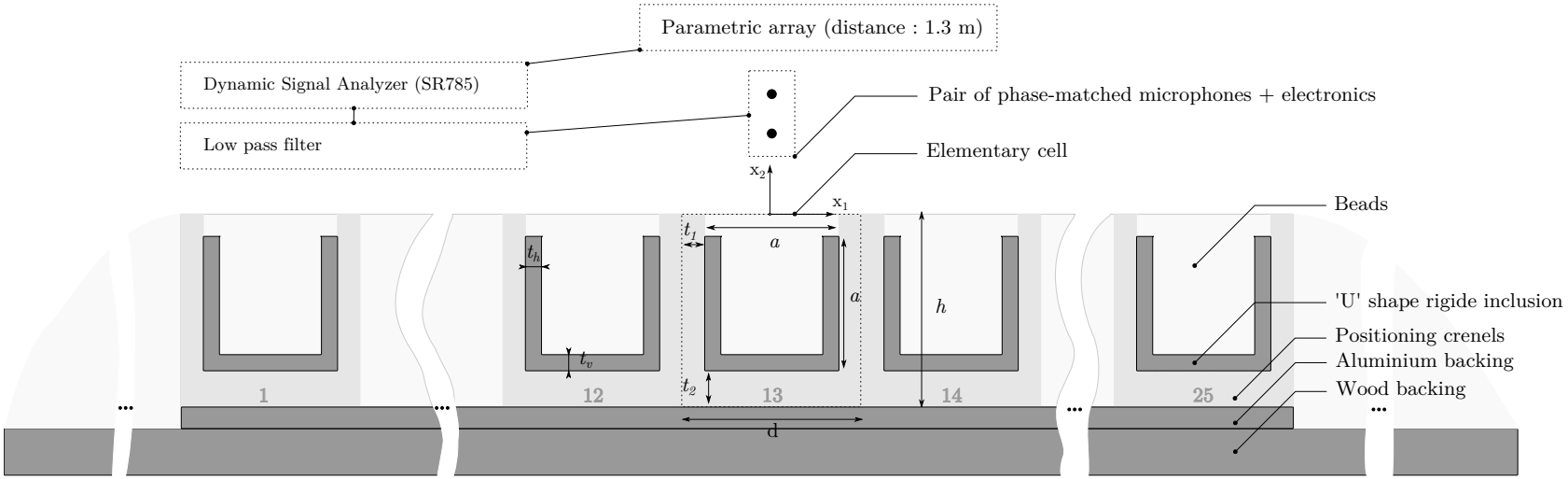
Results

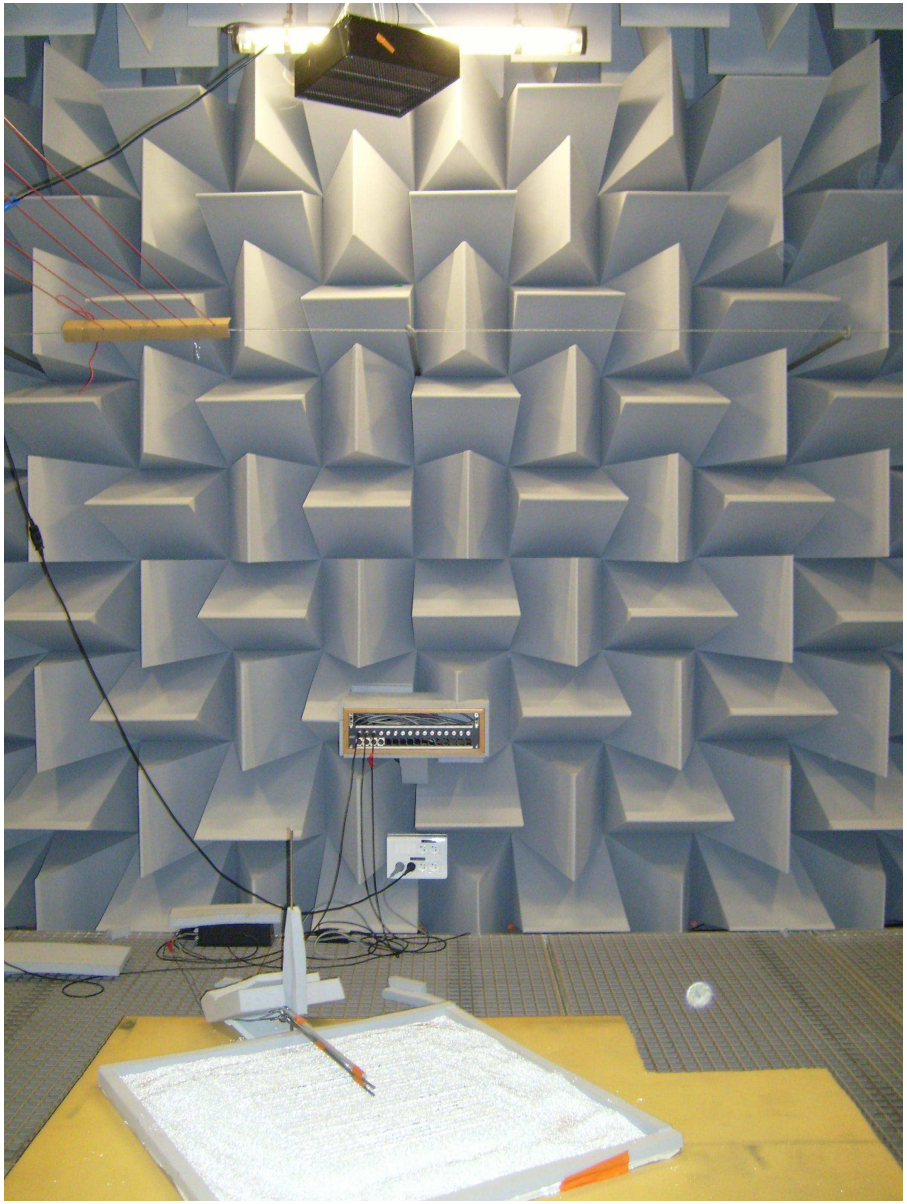
- Experimental validation
- Shape influence
- Conclusions et prospects

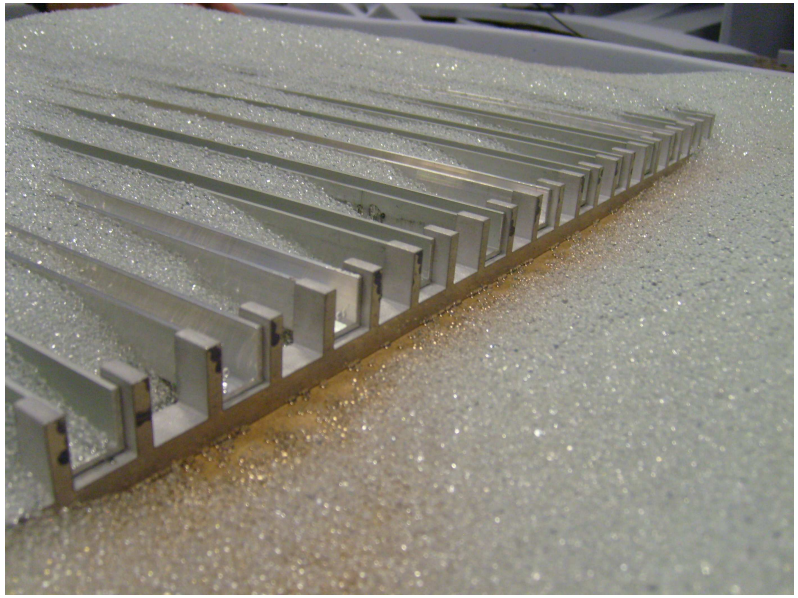
Experimental validation

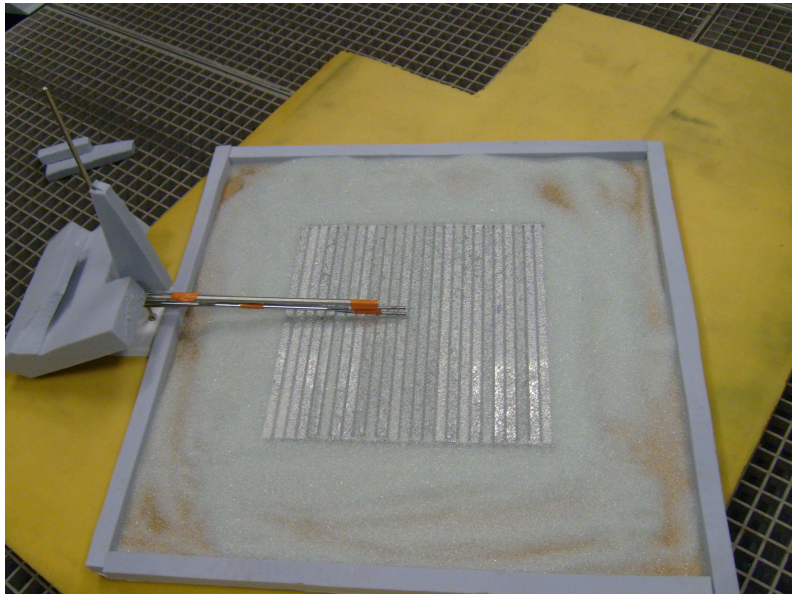
Basic ideas

- Beads (2 mm) offer an easy way to build the porous material
- Tested shapes : 'U', 'C', '○', '[ ]', '□'
- Dimensions : 50x50cm, 2 or 4cm period, total height 2.15cm, 25 periods
- Absorption measurement : pair of phase-matched microphones (local plane wave assumption)



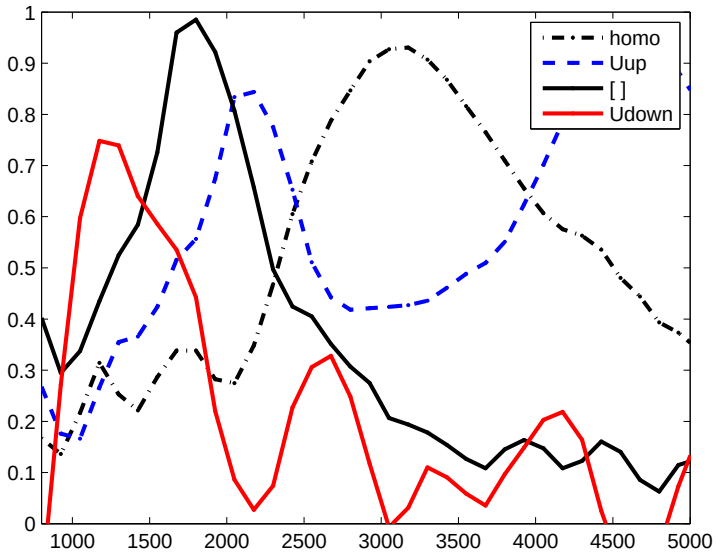




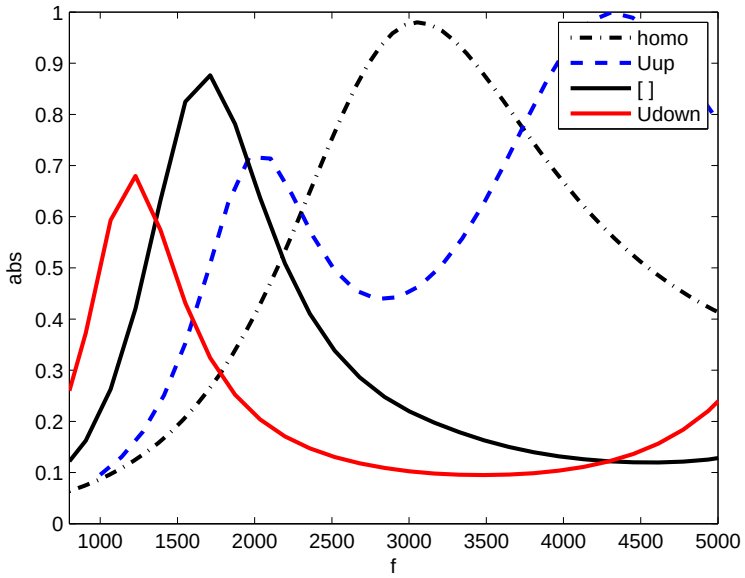


Experimental validation (2)

Effect of rigid inclusion shape on beads absorption



(a) Measurements

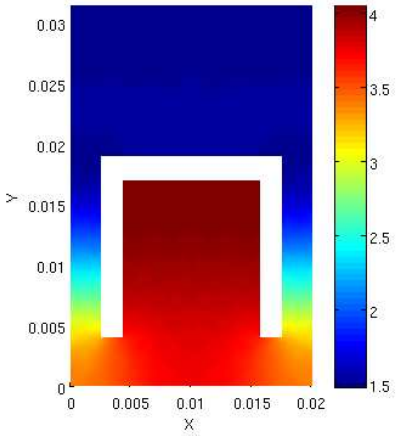


(b) Simulations

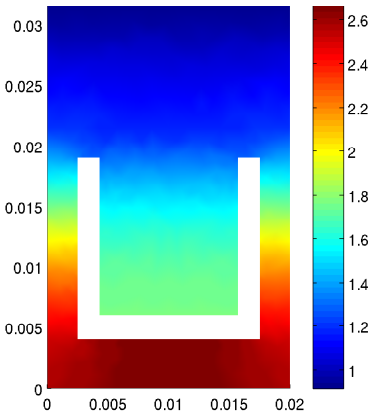
The first absorption peak is pushed in the low frequency range

Inclusion shape

Experimental validation > Pressure modulus on peaks



(a) 1200 Hz



(b) 2041 Hz

(c) 4360 Hz

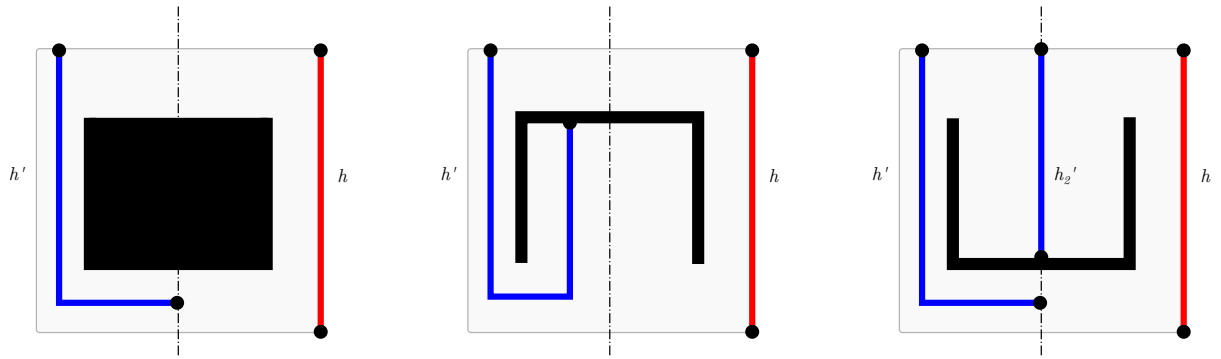
Experimental validation > Simplified model

Assuming the quarter-wave mode remains unchanged

$$f_{\lambda/4} \approx \frac{c_{\mathrm{eq}}}{4h'},$$

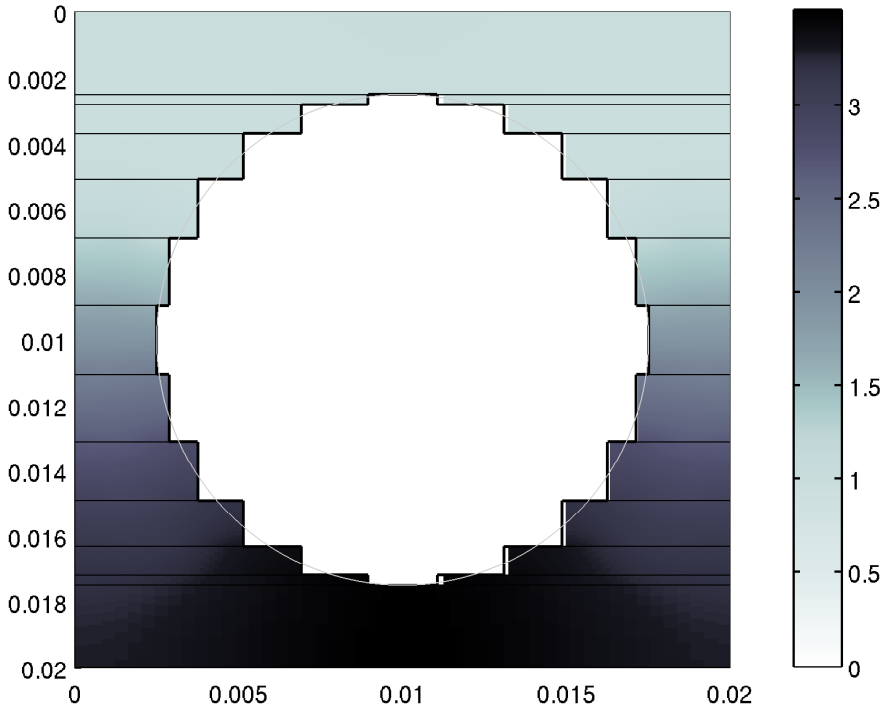
where c_{eq} is the Champoux-Allard-Johnson's wave speed and h' is the effective height of the porous material (see figures below), an approximation of $f_{\lambda/4}$ can be found for simple inclusion shape

Conf.	$f_{\lambda/4}^{\mathrm{exact}}$ (Hz)	h' (mm)	$f_{\lambda/4}^{\mathrm{approx}}$ (Hz)	Error (%)
homogeneous	3050	$h' = h$ =21.5	2950	3.3
■	2041	29.5	2080	1.9
⊓	1200	41.8	1400	16
⊔	2041 – 4360	29.5 – 16.5	2080 – 3940	1.9 – 9.6

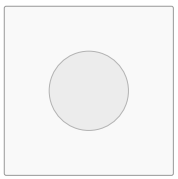


h'/h can be seen as a new tortuosity at mescopic scale.

Shape influence > Fireflex foam



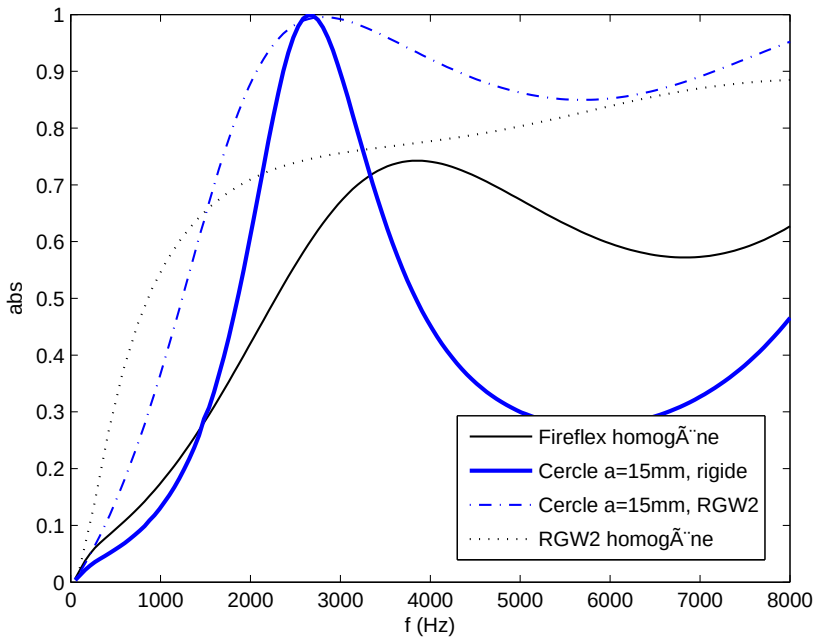
2 cm $\times$ 2cm elementary cell, circular inclusion of radius $a = 1.5$cm [Groby, 2010]



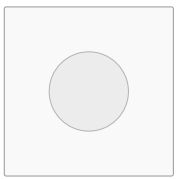
Materials

Foam	ϕ	σ [kNm^{-4}s]	α_{inf}	Λ [μm]	Λ' [μm]
Fireflex	0.95	8900	1.42	180	360
RGW2	0.94	135000	2.1	49	166

Shape influence > Fireflex foam



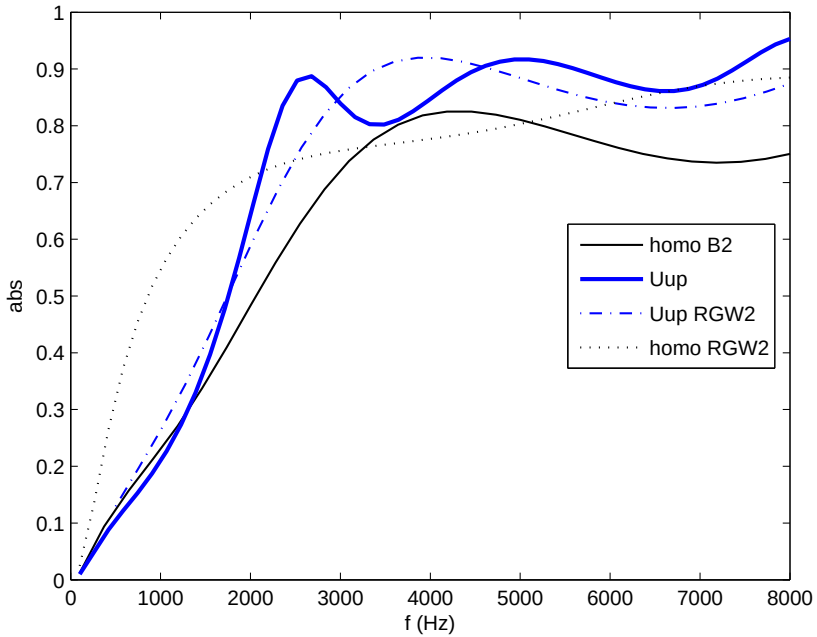
2 cm $\times$ 2cm elementary cell, circular inclusion of radius $a = 1.5$cm [Groby, 2010]



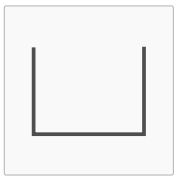
Materials

Foam	ϕ	σ [kNm^{-4}s]	α_{inf}	Λ [μm]	Λ' [μm]
Fireflex	0.95	8900	1.42	180	360
RGW2	0.94	135000	2.1	49	166

Shape influence > metal foam



2 cm $\times$ 2cm elementary cell, U-shape inclusion of side $a = 1.5$cm



Materials

Foam	ϕ	σ [kNm^{-4}s]	α_{inf}	Λ [μm]	Λ' [μm]
metal foam	0.99	6916	1.17	100	245
RGW2	0.94	135000	2.1	49	166

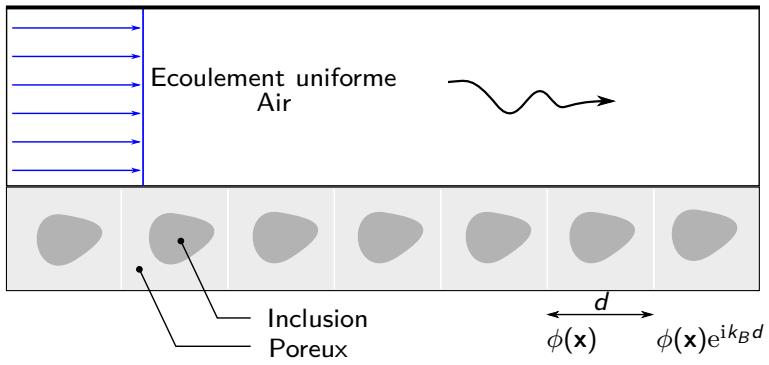
Conclusions et prospects (1)

In this work

1. The interest of non-circular inclusions has been shown computationally and experimentally
2. Measurement have been performed on finite size array
3. A new 'robust' modal method has been developed to tackle periodic inclusions
4. The convergence of the modal series has been improved thanks to the re-expansion scheme [Homentcovchi, 2010]

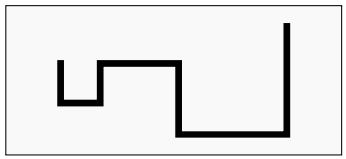
However, it remains some major issues such as

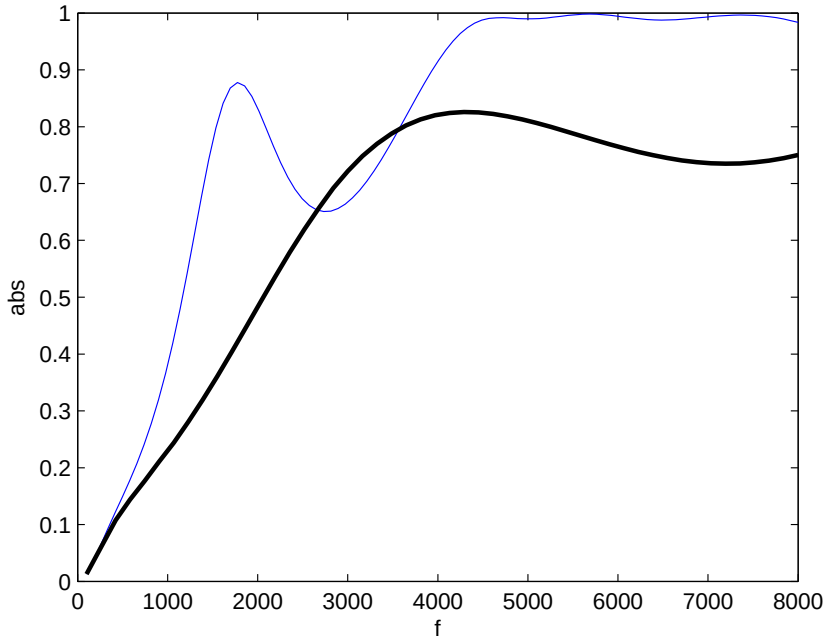
- The shape optimization of the inclusions
- Developed some manufacturing process (Polymerization around inclusion, casting, 3D printer)
- Extend such concepts to waveguide context (work is on going)



Conclusions et prospects (2)

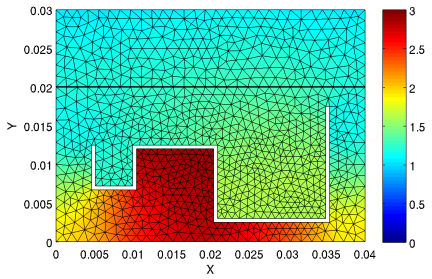
Here are presented some recent results with more complex shape



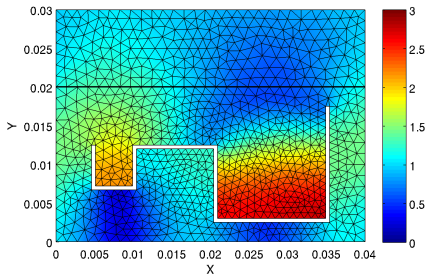


4 cm $\times$ 2cm cell, with snake inclusions vs homogeneous

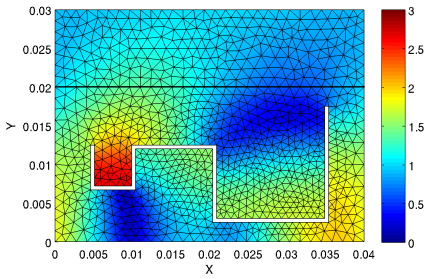
1776 Hz



4648 Hz



5606 Hz



Vi ringrazio per l'attenzione!