Absorption of a rigid frame porous layer with periodic circular inclusions and backed by a periodic grating.

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in collaboration with

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Motivations

Porous material (foam): lack of absorption for audible sound

- Material: properties, microstructure(Perrot *et al*, J.Appl.Phys., 2008)...

- "Propagative" methods:

$\Rightarrow$ Multilayering(Tanneau *et al*, J.Acoust.Soc.Am., 2006)

$\Rightarrow$ Macroscopically inhomogeneous porous medium(De Ryck *et al*, J.Appl.Phys., 2007)

Poster by L. De Ryck(Gautier *et al*, J.Acoust.Soc.Am., 2011)

- "Mixed" methods: homogenization

$\Rightarrow$ Tetrahedra in anechoic room, double porosity media(Boutin *et al*, Int.J.Solids Struct., 1998), ($ISA\beta$(Tournat *et al*, Phys.Rev.E., 2004))

- Methods based on mode excitation

modes excitation $\rightarrow$ local amplification of the field $\rightarrow$ entrapment of energy

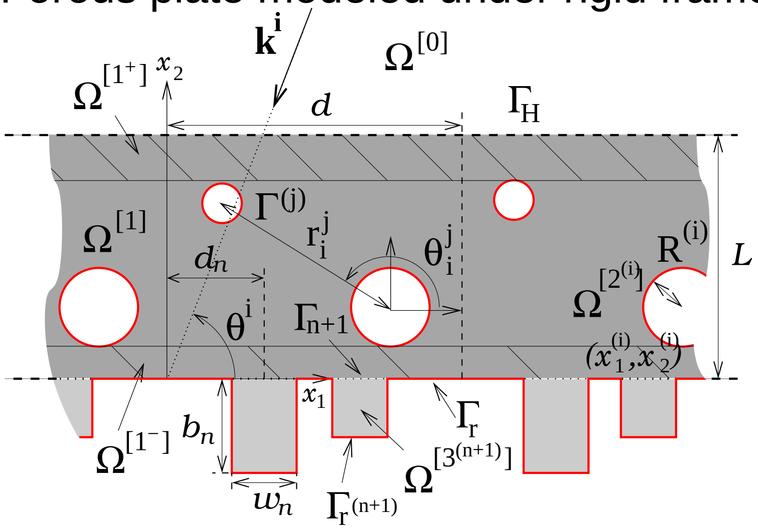
$\Rightarrow$ Volumic heterogeneities: internal entrapment(Groby *et al*, J.Acoust.Soc.Am., 2009, 2011...)

$\Rightarrow$ Surface irregularities: surface entrapment(Allard *et al*, J.Acoust.Soc.Am., 2011)

> Rigid frame porous plate with periodic inclusions backed with multi-irregularities grating

The structured porous material

Porous plate modeled under rigid frame approximation (**Johnson-Champoux-Allard** model)



Look for $p^{[s]}(\mathbf{x}, \omega)$, $\forall \mathbf{x} \in \Omega^{[s]}$ that satisfy

$$\begin{cases} \left(\triangle + (k^{[s]})^2\right) p^{[s]}(\mathbf{x}) = 0, \ \forall \mathbf{x} \in \Omega^{[s]}, \ j = 0, 1, 2^{(n)}, \\ k^{[0]} = \omega / c^{[0]}, \ k^{[s]} = \omega / c^{[s]}(\omega), \ j = 1, 2^{(n)}. \\ \qquad + \\ p^{[0]}(\mathbf{x}) - p^i(\mathbf{x}) \sim \text{ outgoing waves ; } |\mathbf{x}| \to \infty, \ x_2 \geq L \ , \end{cases}$$

wherein $p^i(\mathbf{x}) = A^i e^{\mathrm{i} k_1^i x_1 - \mathrm{i} k_2^{[0]i}(x_2 - a)}$, $k_1^i = -k^{[0]} \cos\left(\theta^i\right)$, $k_2^{[0]i} = k^{[0]} \sin\left(\theta^i\right)$.

Boundary conditions:
$$\begin{cases} [p(\mathbf{x})] = 0, \ \mathbf{x} \in \Gamma_H \\ \left[(\rho(\mathbf{x}))^{-1} \partial_n p(\mathbf{x})\right] = 0, \ \mathbf{x} \in \Gamma_H, \ \Gamma_{(n)} \\ (\rho(\mathbf{x}))^{-1} \partial_n p(\mathbf{x}) = 0, \ \mathbf{x} \in \Gamma_r, \ \Gamma_{r(n)}, \ \Gamma^{(j)} \end{cases} .$$

The field is quasi-periodic (*Floquet condition*) :

$$p((x_1 + nd, x_2)) = p((x_1, x_2)) e^{\mathrm{i} k_1^i nd}, \ \forall \mathbf{x} \in \mathbb{R}^2 \text{ and } \forall n \in \mathbb{Z}.$$

$\Rightarrow$ It is sufficient to determine the field in the unit cell $\mathcal{C}$.

Solution of the problem

Domain decomposition method, field expressions: Block waves

$$p^{[0]}(\mathbf{x}) = \sum_{q\in\mathbb{Z}} \left[A^i e^{-\mathrm{i}k_{2q}^{[0]}(x_2-L)}\delta_{q0} + R_q e^{\mathrm{i}k_{2q}^{[0]}(x_2-L)} \right] e^{\mathrm{i}k_{1q}x_1}, \text{ with } k_{1q} = k_1^i + \frac{2q\pi}{d}.$$

- Application of the B.C. on Γ_H and Γ_0

Linear system of equations: $(\mathcal{A} - \mathcal{C})\,\mathbf{D} - \mathcal{Z}\mathbf{B} = \mathcal{F}$.

$D_m^{(n)}$ amplitude of the pseudo modal representation in the n-th irregularity

$B_l^{(j)}$ amplitude of a diffracted wave by the j-th inclusion

- Application of the Multipole method

Linear system of equations: $(\mathbf{I} - \mathbf{V}\,(\mathbf{S} + \mathbf{Q}))\,\mathbf{B} - \mathbf{VZD} = \mathbf{VF}$.

The final system for the solution of $B_l^{(j)}$ and $D_m^{(n)}$

$$\begin{bmatrix} \mathbf{I} - \mathbf{V}\,(\mathbf{S} + \mathbf{Q}) & -\mathbf{VZ} \\ -\mathcal{Z} & \mathcal{A} - \mathcal{C} \end{bmatrix} \begin{pmatrix} \mathbf{B} \\ \mathbf{D} \end{pmatrix} = \begin{pmatrix} \mathbf{VF} \\ \mathcal{F} \end{pmatrix}.$$

Reflected fields & Acoustic properties

$$p^{[0]}(\mathbf{x}) = A^i e^{\mathrm{i}\left(k_1^i x_1 - k_2^{[0]i}(x_2-L)\right)} + A^i \frac{\alpha^{[0]i}\cos\left(k_2^{[1]i}L\right) + \mathrm{i}\alpha^{[1]i}\sin\left(k_2^{[1]i}L\right)}{D^i} e^{\mathrm{i}k_1^i x_1 + \mathrm{i}k_2^{[0]i}(x_2-L)}$$

$$+ \sum_{q\in\mathbb{Z}}\sum_{n\in\mathcal{N}} \frac{\mathrm{i}w_n e^{-\mathrm{i}k_{1q}(d_n - w_n/2)}}{dD_q} \sum_{m=0}^{\infty} D_m^{(n)} \alpha_m^{[2^{(n)}]} \sin\left(k_{2m}^{[2^{(n)}]} b_n\right) I_{qm}^{-(n)} e^{\mathrm{i}k_{1q}x_1 + \mathrm{i}k_{2q}^{[0]}(x_2-L)}$$

$$+ \sum_{q\in\mathbb{Z}}\sum_{j\in\mathcal{N}^c}\sum_{l\in\mathbb{Z}} \frac{4(-\mathrm{i})^l \alpha_q^{[1]}}{dk_{2q}^{[1]} D_q} B_l^{(j)} \cos\left(k_{2q}^{[1]} x_2^{(j)} - l\theta_q\right) e^{-\mathrm{i}k_{1q}x_1^{(j)}} e^{\mathrm{i}k_{1q}x_1 + \mathrm{i}k_{2q}^{[0]}(x_2-L)},$$

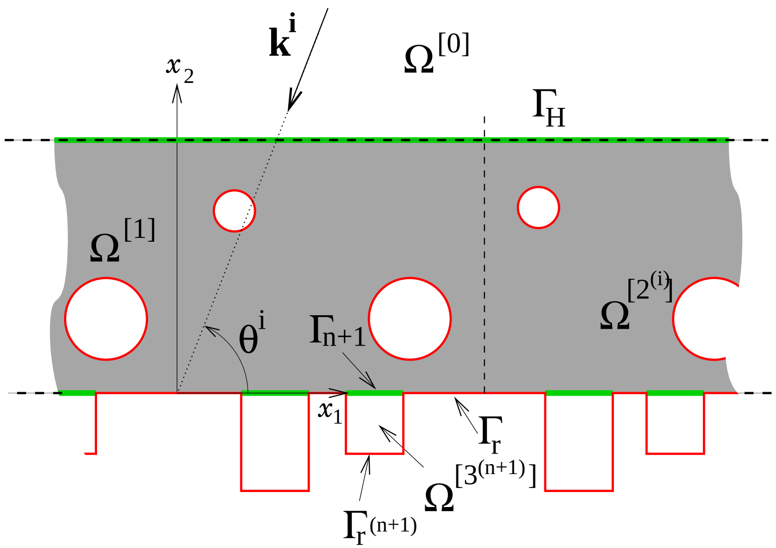
with $D_q = \alpha_q^{[0]}\cos\left(k_{2q}^{[1]}L\right) - \mathrm{i}\alpha_q^{[1]}\sin\left(k_{2q}^{[1]}L\right)$ (Pekeris mode - Love mode)

The developed form of the energy conservation relation is

$$1 = \mathcal{A} + \mathcal{R}\,,$$

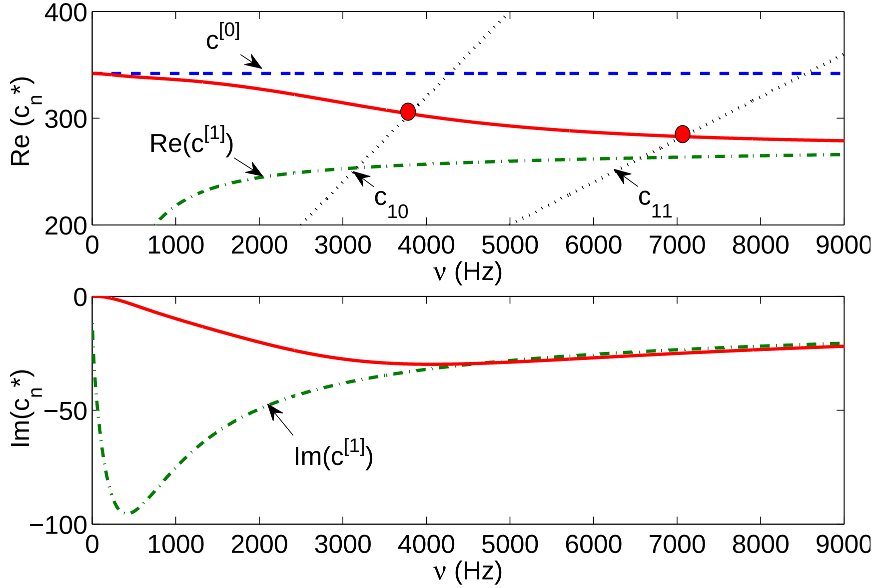
with $\mathcal{R} = \sum_{q\in\mathbb{Z}} \frac{\mathrm{Re}\left(k_{2q}^{[0]}\right)}{k_2^{[0]i}} \frac{\|R_q\|^2}{\|A^i\|^2}$.

$\Rightarrow \mathcal{A} = 1 - \mathcal{R}\,.$



The modified modes of the plate

$L = 2$cm, Fireflex: $\phi = 0.95$, $\alpha_\infty = 1.42$, $\Lambda = 180$ μm, $\Lambda = 360$ μm, $\sigma = 8900$ Nsm^{-4} ($\nu_c \approx 781$Hz)



$$\alpha_q^{[0]} \cos\left(k_{2q}^{[1]} L\right) - \mathrm{i}\alpha_q^{[1]} \sin\left(k_{2q}^{[1]} L\right) = 0$$

Example for $d = 8$ cm

Mode : Re$\left(c_{1(n)}\right) = c_{1q} = \omega / k_{1q}$

Associate attenuation: Im $\left(c_{1(n)}\right)$

Modified modes of the plate are excited even for plane incident waves

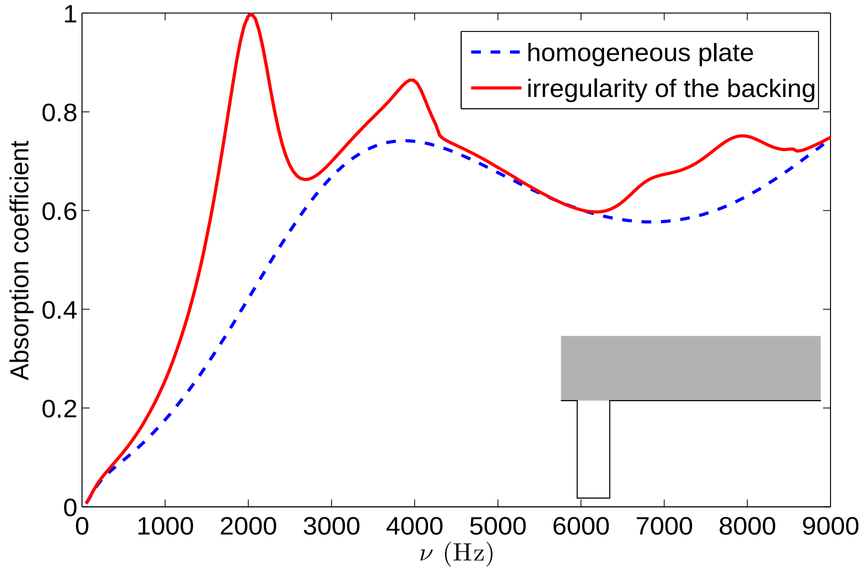
Associated waves: propagative waves in the plate & evanescent waves in air

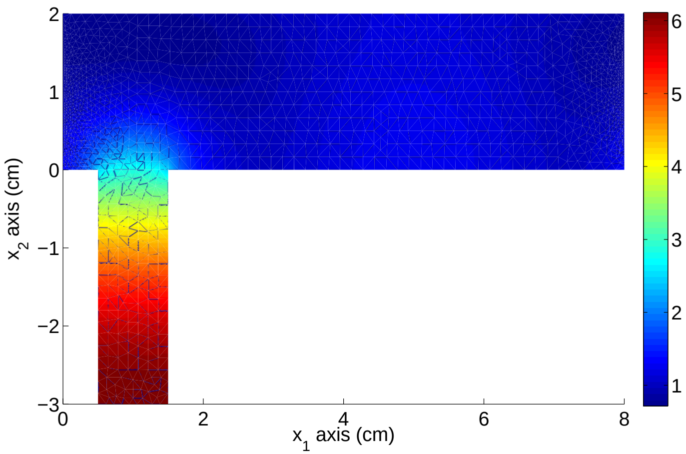
$\Rightarrow$ entrapment of the energy in the plate $\rightarrow$ increase of the absorption.

Trapped mode of the cavity

One irregularity $b \times w = 3$ cm $\times\ 1$ cm , $L = 2$ cm, $d = 8$ cm , $\theta^i = \pi/2$

$$\boxed{\cos\left(k_{2m}^{[3]} b\right) = 0} \Rightarrow \nu_t^i \approx c^{[3]}/4b \approx 2000 \text{ Hz}$$





Snapshot of the pressure field at ν_t^i

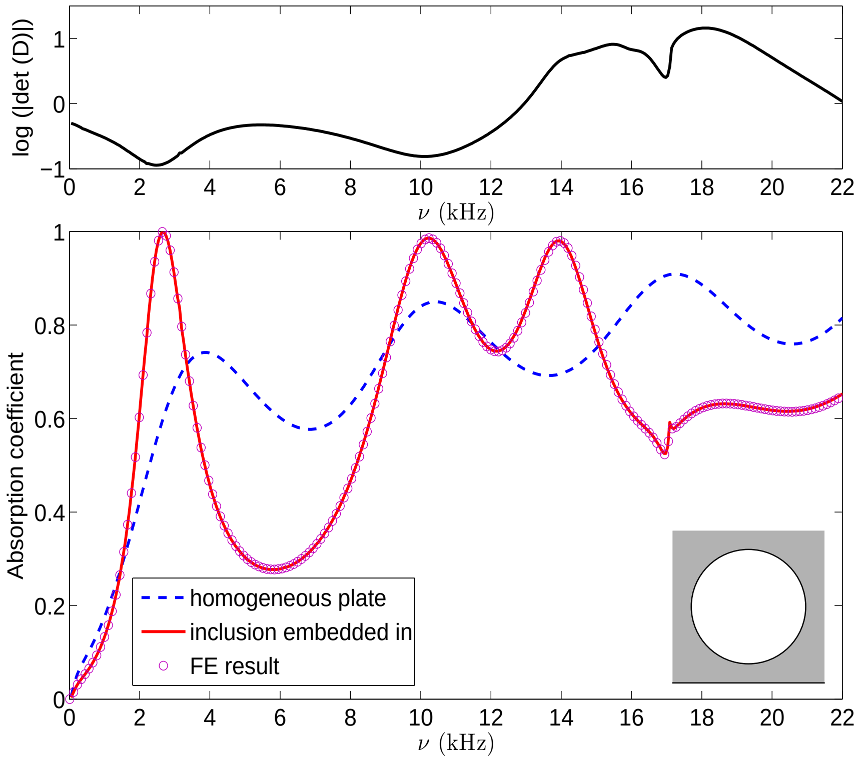
ν_t^i is all the smaller that b is large

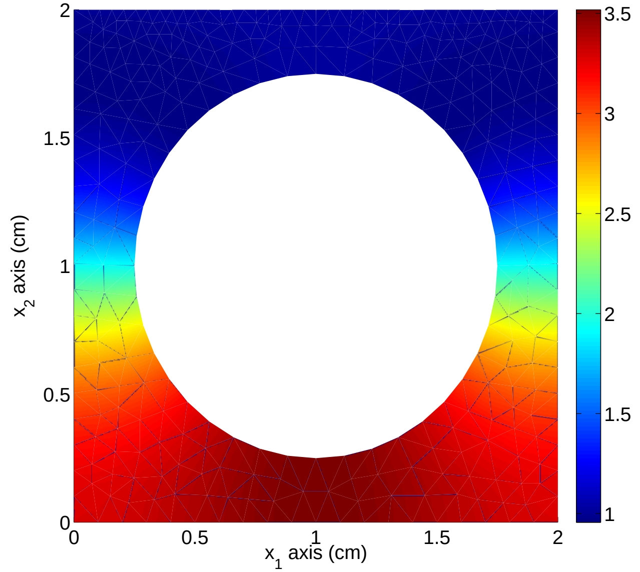
$\Rightarrow$ energy entrapment in the cavity (almost independant from θ^i)

J.-P. Groby *et al.*, J. Acoust. Soc. Am., 127: 2865-2874, 2010

Trapped mode associated with the inclusions

One inclusion $r = 0.75$ cm , $L = 2$ cm, $d = 2$ cm , $\theta^i = \pi/2$





Sanpshot of the pressure field at ν_t^c

ν_t^c is all the smaller that $x_2^{(j)}$ is large

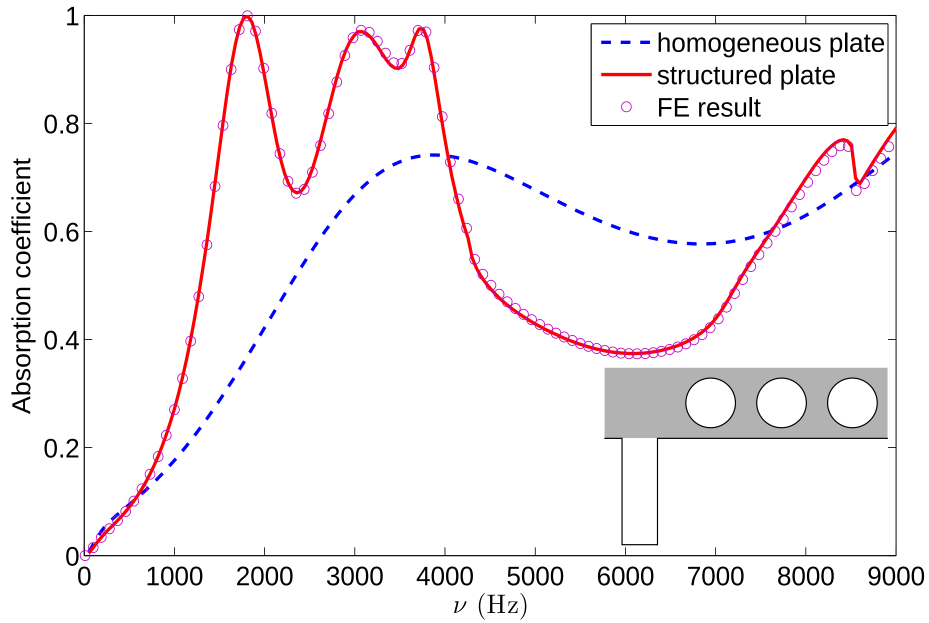
$\Rightarrow$ energy entrapment between the inclusions and the plate (almost independant from θ^i)

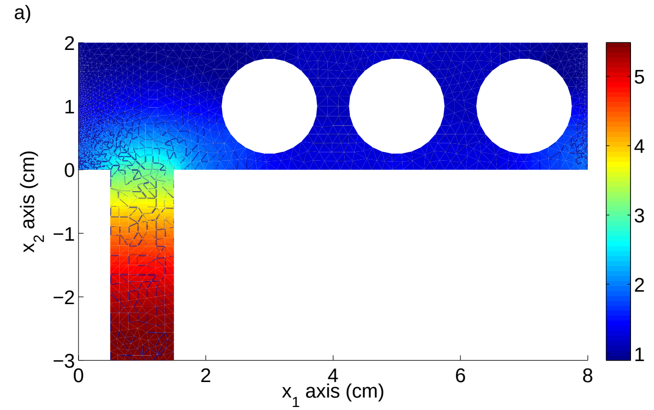
J.-F. Allard *et al.*, J. Acoust. Soc. Am., 129: 1696-1706, 2011

J.-P. Groby *et al.*, to appear J. Acoust. Soc. Am.

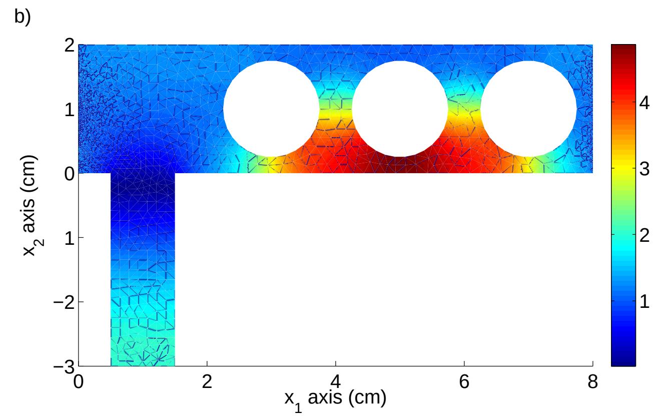
1 irregularity and 3 inclusions

One irregularity $b\times w=3$ cm$\times 1$ cm and 3 inclusions $r=0.75$ cm , $L=2$ cm, $d=8$ cm , $\theta^i=\frac{\pi}{2}$



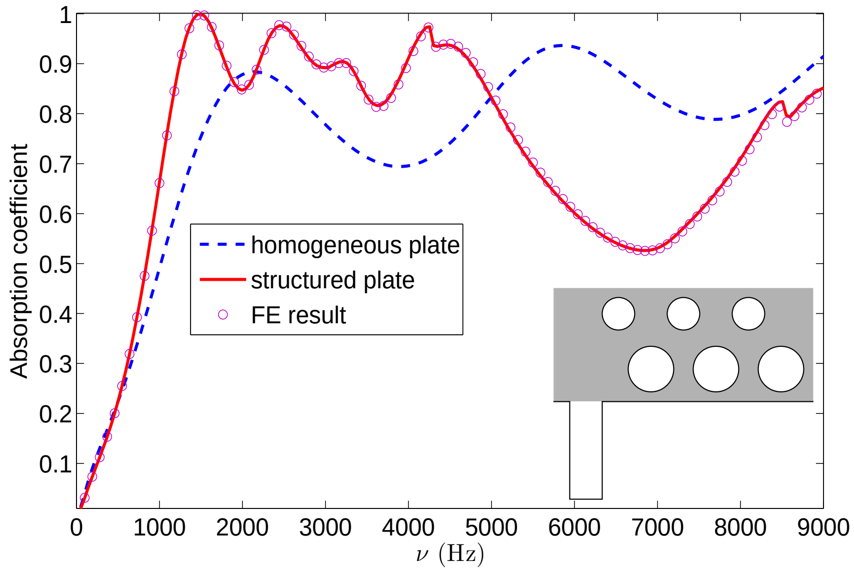


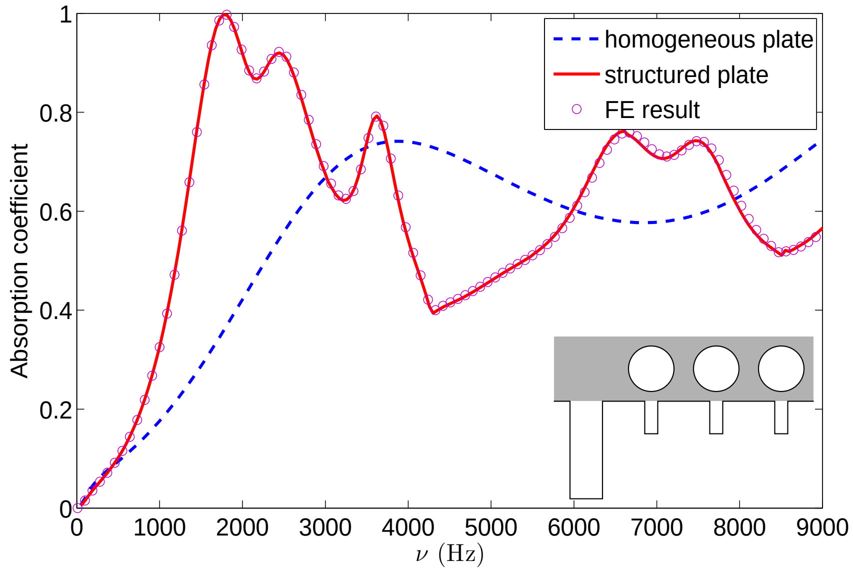
Snapshot of the pressure field at ν_t^c



Snapshot of the pressure field at ν_t^i

More complicated unit cell





J.-P. Groby *et al.*, J. Acoust. Soc. Am., 129: 3035-3046, 2011

Conclusion&Perspectives

- Satisfying results

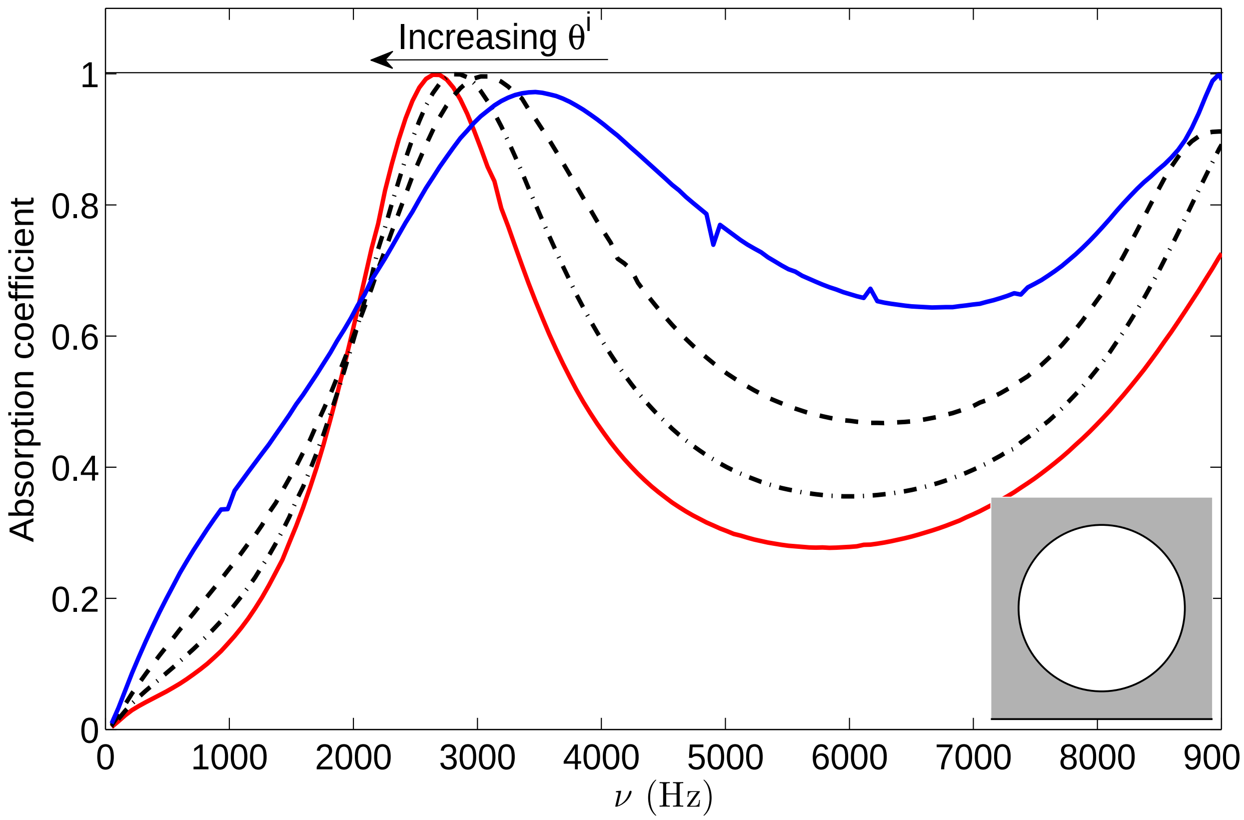
  $\Rightarrow$ entrapment of the energy inside the strcuture.

  - influence of the shape and properties of the irregularities and inclusions (following presentation by B. Nennig)
  - resonating inclusions: Helmholtz resonator (presentation by C. Lagarrigue)
  - influence of the properties of the porous layer
  - manufacturing? $\Rightarrow$ fiber like material
  - what about the moving frame?
- 3D configurations

Thank you!

Merry Christmas

Influence of the incident field



$\theta^i = \pi/2$ (—), $\theta^i = \pi/3$ ($- \cdot -$), $\theta^i = \pi/4$ ($- - -$), and $\theta^i = \pi/6$ (—)

Application of the B.C. on Γ_H and Γ_0

Separation of variables technique, outgoing wave condition and Floquet's theorem:

- $$p^{[0]}(\mathbf{x}) = \sum_{q\in\mathbb{Z}} \left[A^i e^{-\mathrm{i}k_{2q}^{[0]}(x_2-L)}\delta_{q0} + R_q e^{\mathrm{i}k_{2q}^{[0]}(x_2-L)} \right] e^{\mathrm{i}k_{1q}x_1},$$
- $$p^{[1]}(\mathbf{x}) = \sum_{q\in\mathbb{Z}} \left(f_q e^{-\mathrm{i}k_{2q}^{[1]}x_2} + g_q e^{\mathrm{i}k_{2q}^{[1]}x_2} \right) e^{\mathrm{i}k_{1q}x_1}$$
$$+\sum_{q\in\mathbb{Z}}\sum_{j\in\mathcal{N}^c}\sum_{l\in\mathbb{Z}} K_{ql}^{\pm} B_l^{(j)} e^{\mathrm{i}\left(k_{1q}\left(x_1-x_1^{(j)}\right)\pm k_{2q}^{[1]}\left(x_2-x_2^{(j)}\right)\right)}, \forall \mathbf{x}\in\Omega^{[1^{\pm}]}$$

with $k_{1q}=k_1^i+\frac{2q\pi}{d}$, $k_{2q}^{[j]}=\sqrt{(k^{[j]})^2-(k_{1q})^2}$ such that Re$\left(k_{2q}^{[j]}\right)\geq 0$ and Im$\left(k_{2q}^{[j]}\right)\geq 0$, $j=0,1$.

- $$p^{[2^{(n)}]}(\mathbf{x}) = \sum_{m=0}^{\infty} D_m^{(n)} \cos\left(k_{1m}^{[2^{(n)}]}(x_1-d_n+w_n/2)\right)\cos\left(k_{2m}^{[2^{(n)}]}(x_2+b_n)\right),$$

wherein $k_{1m}^{[2^{(n)}]}=\frac{m\pi}{w_n}$, $k_{2m}^{[2^{(n)}]}=\sqrt{(k^{[2]})^2-(k_{1m}^{[2^{(n)}]})^2}$, with Re$\left(k_{2m}^{[2^{(n)}]}\right)\geq 0$ and Im$\left(k_{2m}^{[2^{(n)}]}\right)\geq 0$.

After introduction into the B.C. on Γ_L and $\Gamma_{(n)}\bigcup\Gamma_r$, use of the appropriate projection

Linear system of equations: $(\mathcal{A}-\mathcal{C})\,\mathbf{D}-\mathcal{Z}\mathbf{B}=\mathcal{F}$.

Application of the multipole method

In the vicinity of the J-th inclusion:

- $$p^{[1]}(\mathbf{r}_J) = \sum_{L\in\mathbb{Z}} \left(B_L^{(J)} \mathsf{H}_L^{(1)}\left(k^{[1]} r_J\right) + A_L^{(J)} \mathsf{J}_L\left(k^{[1]} r_J\right)\right) e^{\mathrm{i}L\theta_J}$$

wherein $$A_L^{(J)} = \sum_{l\in\mathbb{Z}} S_{L-l} B_l^{(J)} + \sum_{j\neq J}\sum_{l\in\mathbb{Z}} S_{L-l}^{(J,j)} B_l^{(j)} + \sum_{j\in\mathcal{N}^c}\sum_{l\in\mathbb{Z}}\sum_{q\in\mathbb{Z}} Q_{qLl}^{(J,j)} B_l^{(j)} + \sum_{n\in\mathcal{N}^i}\sum_{m\in\mathbb{Z}}\sum_{q\in\mathbb{Z}} Z_{qLm}^{(J,n)} D_m^{(n)} + \sum_{q\in\mathbb{Z}} F_{qL}^{(J)}.$$

After introduction into the B.C. on $\Gamma^{(J)}$, use of the appropriate projection: $B_l^{(J)} = V_L^{(J)} A_l^{(J)}$

Linear system of equations: $(\mathbf{I} - \mathbf{V}(\mathbf{S}+\mathbf{Q}))\mathbf{B} - \mathbf{VZD} = \mathbf{VF}$

The final system for the solution of $B_l^{(j)}$ and $D_m^{(n)}$

$$\begin{bmatrix} \mathbf{I} - \mathbf{V}(\mathbf{S}+\mathbf{Q}) & -\mathbf{VZ} \\ -\boldsymbol{\mathcal{Z}} & \boldsymbol{\mathcal{A}} - \boldsymbol{\mathcal{C}} \end{bmatrix} \begin{pmatrix} \mathbf{B} \\ \mathbf{D} \end{pmatrix} = \begin{pmatrix} \mathbf{VF} \\ \boldsymbol{\mathcal{F}} \end{pmatrix}$$