Sensitivity analysis of two “equivalent fluid” models

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Motivations

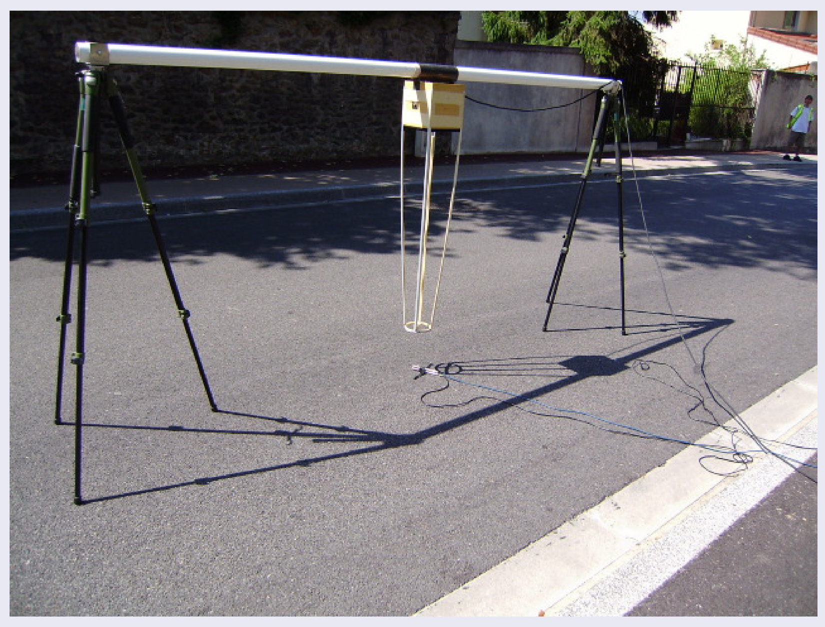
- Clogging of porous road surfaces ⇒ loss of acoustic absorption and security problems
- Characterizing the loss of acoustic absorption ⇒ assessing the extent of clogging phenomenon
- How to relate clogging to acoustic performances :

Equivalent fluid models

Microstructure related parameters	In situ measurable acoustic quantities
σ, Φ, α_∞, Λ_v, Λ_t, etc	Complex surface impedance Z (2 equations)
n unknowns, n > 2 in most models	no access to equivalent density ρ_{eq} and incompressibility K_{eq}

Inversion of the chosen model to assess the clogging phenomenon

↪ A criterion is needed to distinguish :
- which parameters may be neglected or not,
- where : on which frequency ranges,
- when : under which conditions (thickness known ? parameters variation ranges ?).



Measurement set up for a possible in situ measurement method (the transfer function method)

Analysed models

- Biot-Allard model associated with Zwikker & Kosten model [1, 2, 3]

3 parameters : resistivity σ, porosity Φ and tortuosity α_∞, and a shape factor s set to 1.

$$\begin{cases} \rho_{eq}(\omega) = \frac{\rho_0\alpha_\infty}{\Phi} - \frac{j\sigma}{\omega}F(\lambda) \\ K_{eq}(\omega) = \frac{\gamma P_0}{\Phi}\left[1 + 2(\gamma-1)\frac{T(\sqrt{N_{Pr}}\lambda\sqrt{-j})}{\sqrt{N_{Pr}}\lambda\sqrt{-j}}\right]^{-1} \end{cases}$$

with $\omega = 2\pi f$ the pulsation, ρ_0 the air density, F the correction function for viscosity defined by Biot, T the ratio of Bessel functions of the first kind of order 1 and 0, and $\lambda = s\sqrt{\frac{8\alpha_\infty\rho_0\omega}{\sigma\Phi}}$.

- Johnson-Champoux-Allard model [4, 5]

5 parameters : σ, Φ, α_∞ and viscous and thermal characteristic lengths Λ_v and Λ_t.

$$\begin{cases} \rho_{eq}(\omega) = \frac{\alpha_\infty\rho_0}{\Phi}\left[1 - j\frac{\sigma\Phi}{\omega\rho_0\alpha_\infty}\sqrt{1 + j\frac{4\alpha_\infty{}^2\eta\rho_0\omega}{\sigma^2\Lambda^2\Phi^2}}\right] \\ K_{eq}(\omega) = \frac{\gamma P_0/\Phi}{\gamma - (\gamma-1)\left[1 - j\frac{8\eta}{\Lambda_t{}^2B^2\omega\rho_0}\sqrt{1 + j\frac{\rho_0\omega B^2\Lambda_t{}^2}{16\eta}}\right]^{-1}} \end{cases}$$

with η the dynamic viscosity of air.

- Relations to wave number k, characteristic impedance Z_c and measured surface impedance Z

$$\begin{cases} Z_c(\omega) = \sqrt{\rho_{eq}(\omega) \times K_{eq}(\omega)} \\ k(\omega) = \omega\sqrt{\frac{\rho_{eq}}{K_{eq}}} \end{cases} \qquad Z(\omega) = -j\, Z_c \operatorname{cotan}(k\, e)$$

with e the material thickness.

Sensitivity index

- Definition of the sensitivity index :
  S_i : sensitivity index of Y to parameter X_i

Input $(X_1, ...X_i, ..., X_n)$ → Model function f → Output Y

$$S_i = \frac{\mathrm{V}(\mathrm{E}[Y|X_i])}{\mathrm{V}(Y)} = \frac{\mathrm{E}[\mathrm{E}[Y|X_i]] - \mathrm{E}[Y]^2}{\mathrm{V}(Y)} = \frac{U_i - \mathrm{E}[Y]^2}{\mathrm{V}(Y)}$$

with V the variance and E the expectation of a random variable.

- Estimating S_i thanks to Monte Carlo methods :
  N-samples of each of the inputs $(X_1, ..., X_n)$ are considered.

$$\hat{f}_0 = \hat{\mathrm{E}}[Y] = \frac{1}{N}\sum_{k=1}^{N} f(x_{k1}, x_{k2}, .., x_{kn})$$

$$\hat{\mathrm{V}} = \hat{\mathrm{V}}(Y) = \frac{1}{N}\sum_{k=1}^{N} f^2(x_{k1}, x_{k2}, .., x_{kp}) - E[Y]^2$$

- Difficulty to estimate a conditional expectation:
  ↪ Use of Sobol method [6]

A second N-sample of the input X_i is introduced to evaluate the last term :

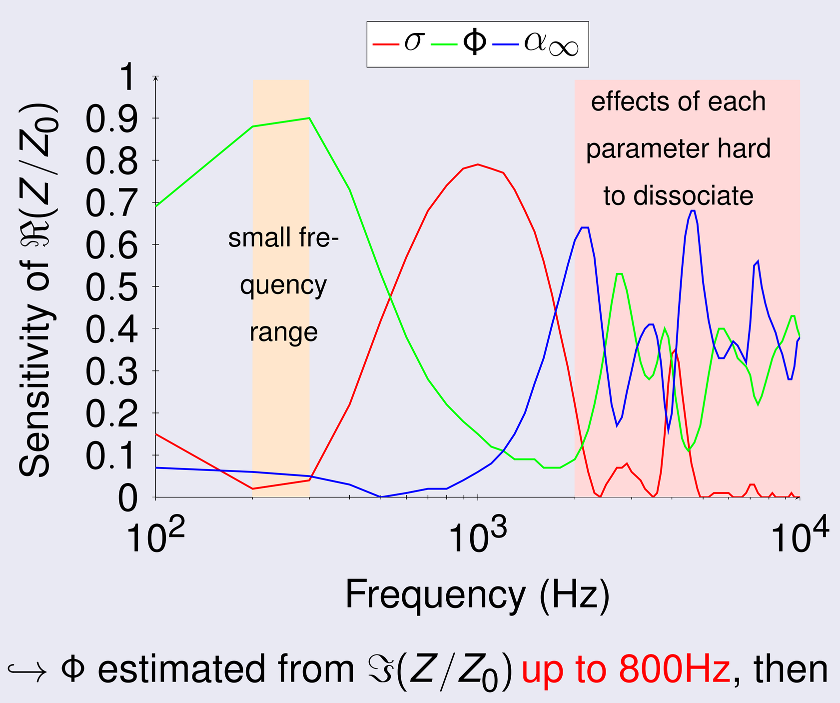
$$\hat{\mathrm{U}}_i = \hat{\mathrm{E}}[\mathrm{E}[Y|X_i]] = \frac{1}{N}\sum_{k=1}^{N} f(x_{k1}^{(1)}, ..., x_{k(i-1)}^{(1)}, x_{ki}^{(1)}, x_{k(i+1)}^{(1)}, ..., x_{kp}^{(1)}) \times f(x_{k1}^{(2)}, ..., x_{k(i-1)}^{(2)}, x_{ki}^{(1)}, x_{k(i+1)}^{(2)}, ..., x_{kp}^{(2)})$$

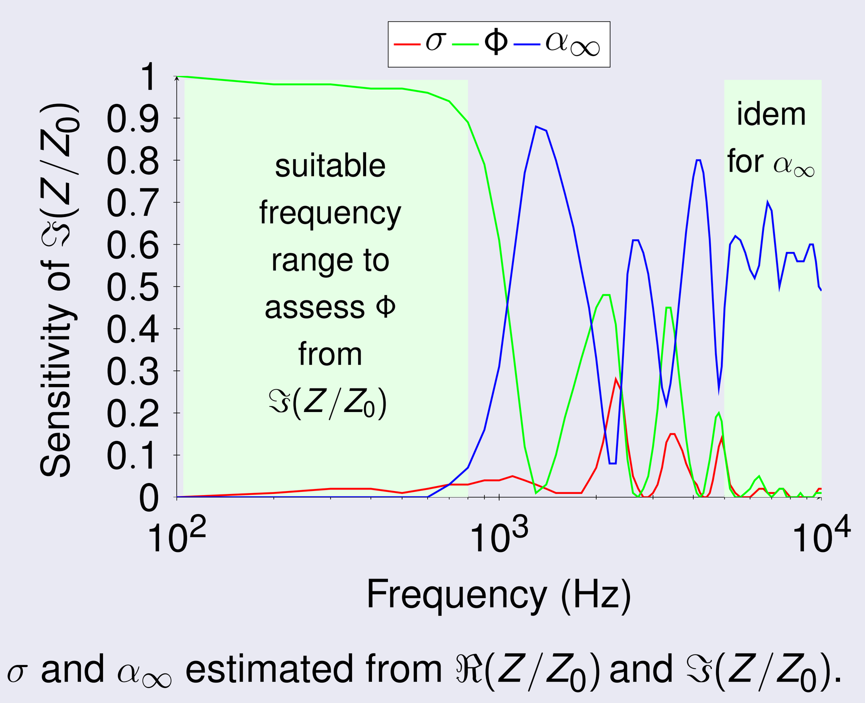
↪ A sensitivity index calculated with parameters varying simultaneously.

Sensitivity analysis of Biot-Allard model

- **If thickness is known**

Variation ranges (from porous road surface literature [7, 8, 9]) :
Φ : $[0.08 - 0.22]$; σ : $[15000 - 85000]$Nm^{-4}s ; α_∞ : $[1.3 - 2.7]$; thickness $e = 4$ cm.

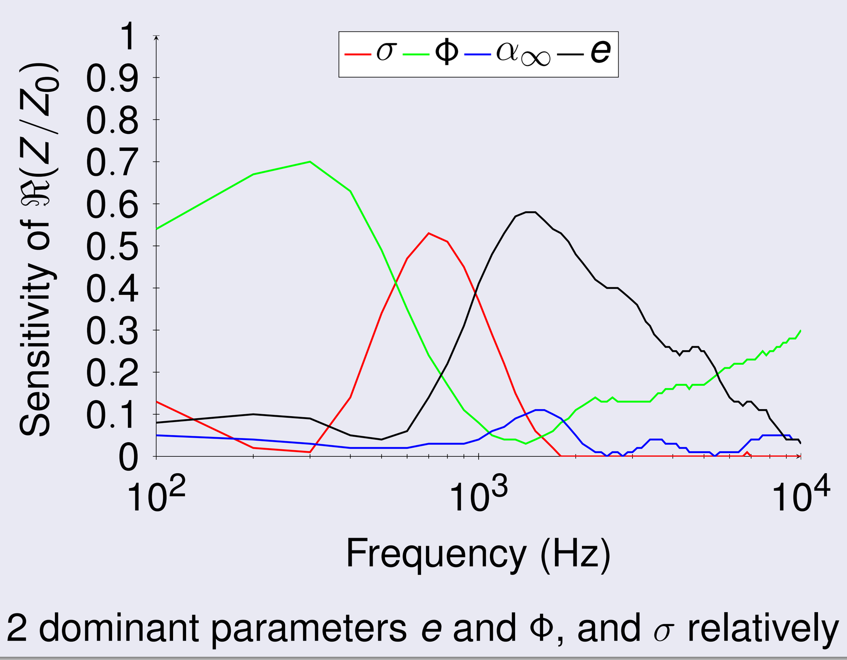


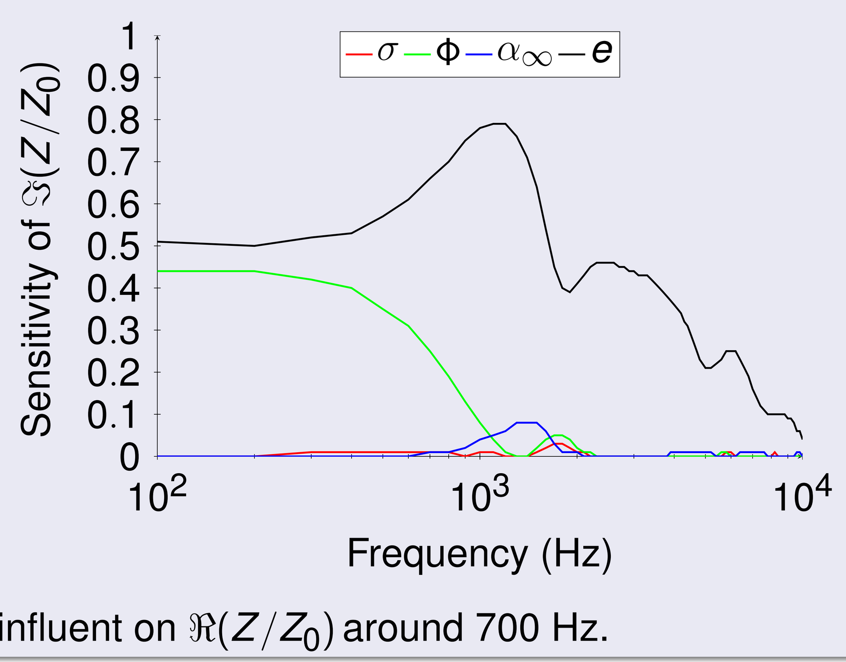


↪ Φ estimated from $\Im(Z/Z_0)$ up to 800Hz, then σ and α_∞ estimated from $\Re(Z/Z_0)$ and $\Im(Z/Z_0)$.

- **If thickness is unknown**

Variation range for thickness : $[2 - 6]$cm.



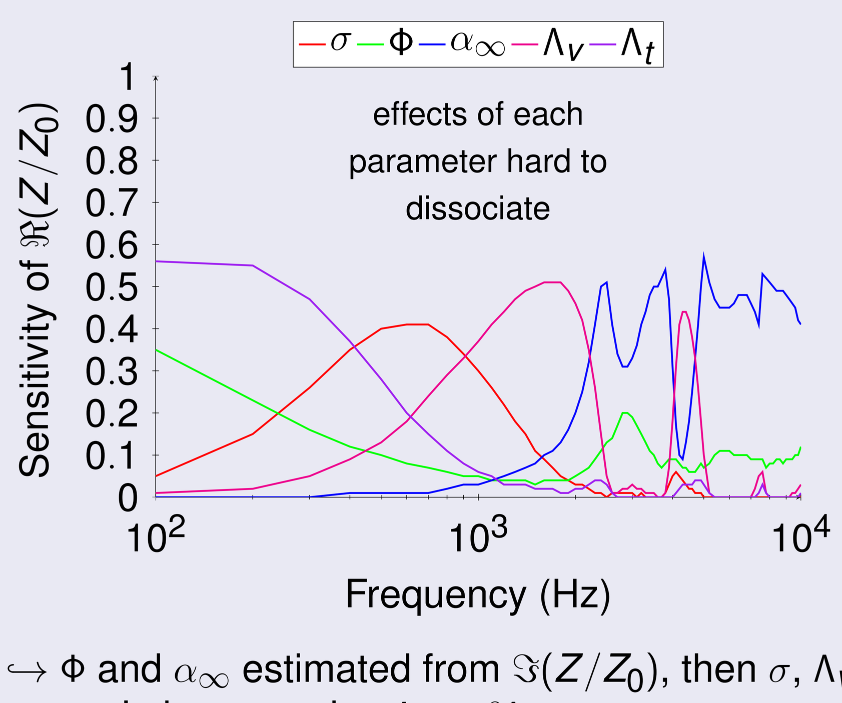


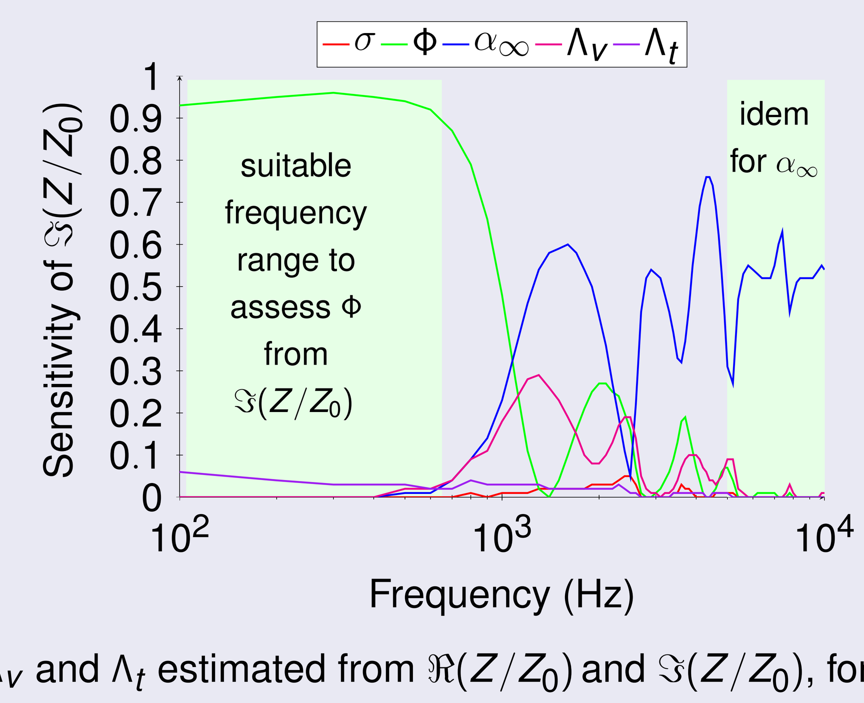
2 dominant parameters e and Φ, and σ relatively influent on $\Re(Z/Z_0)$ around 700 Hz.

Sensitivity analysis of Johnson-Champoux-Allard model

- **If thickness is known**

Variation ranges : Φ : $[0.08 - 0.22]$; σ : $[15000 - 85000]$Nm^{-4}s ; α_∞ : $[1.3 - 2.7]$;
Λ_t : $[200 - 2400]\mu m$; Λ_v : $[60\mu m - \Lambda_t]$; thickness $e = 4$ cm.

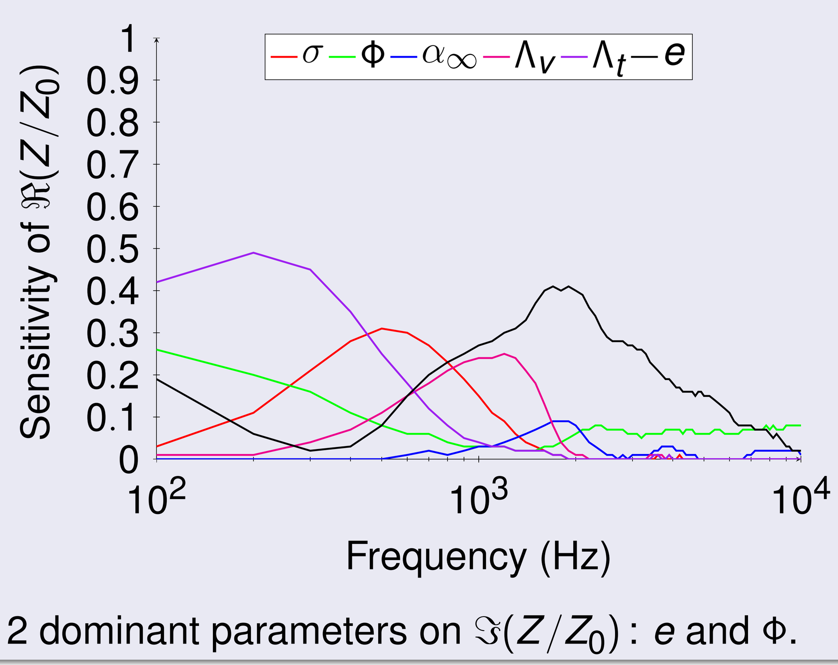


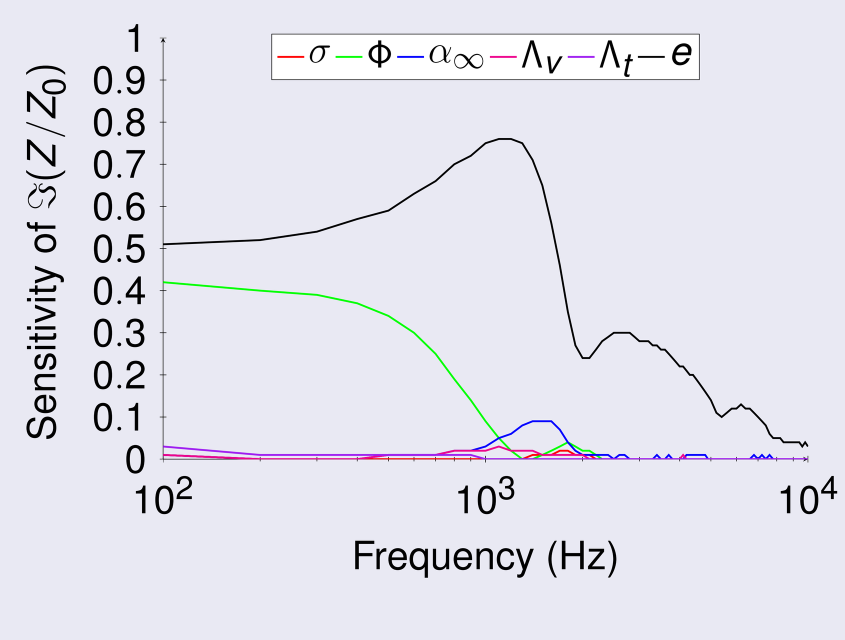


↪ Φ and α_∞ estimated from $\Im(Z/Z_0)$, then σ, Λ_v and Λ_t estimated from $\Re(Z/Z_0)$ and $\Im(Z/Z_0)$, for example by assuming $\Lambda_t = \beta\Lambda_v$.

- **If thickness is unknown**

Variation range for thickness : $[2 - 6]$cm.





2 dominant parameters on $\Im(Z/Z_0)$: e and Φ.

Conclusions

- A qualitative and quantitative criterion used to help to inverse a model.
- This sensitivity analysis allows to establish an inversion procedure.
- Importance of thickness when unknown in situ
  ⇒ thickness measurement + acoustic measurement may be required.
- A measurement method efficient at low and high frequencies required or 2 measurements methods.
- Importance of the choice of the variation range of the parameters.

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