Measuring low frequencies parameters of porous materials having a rigid frame.

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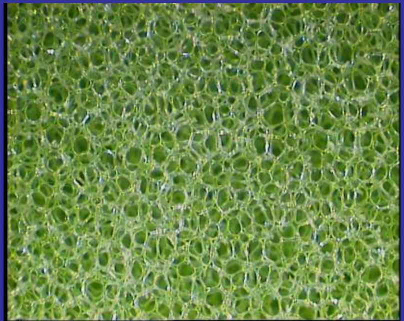
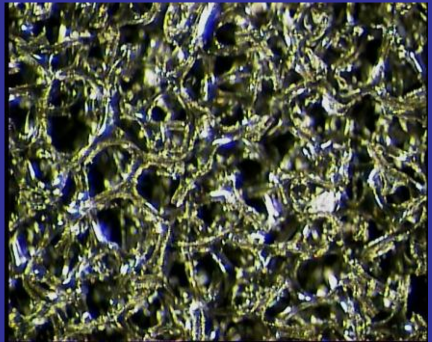
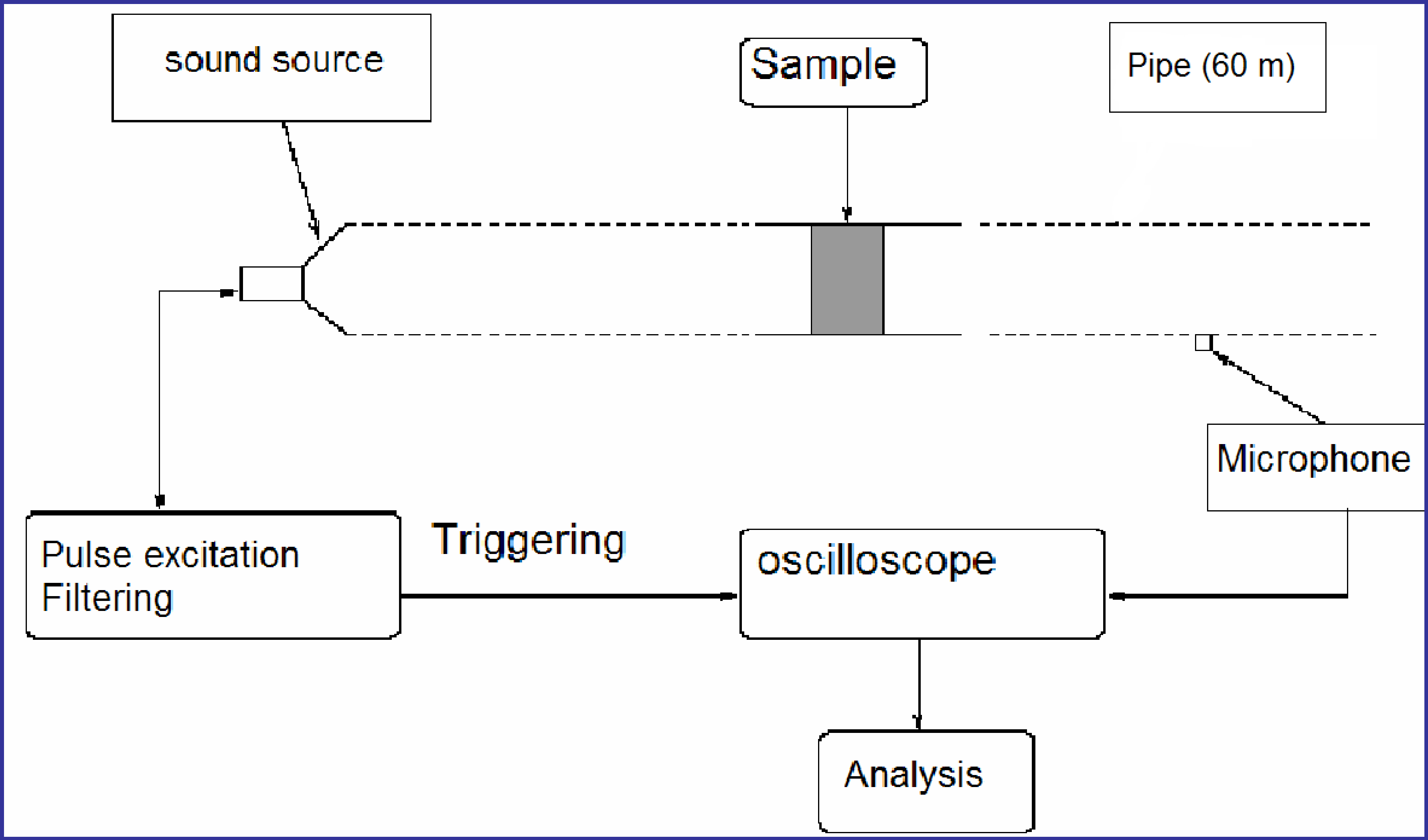
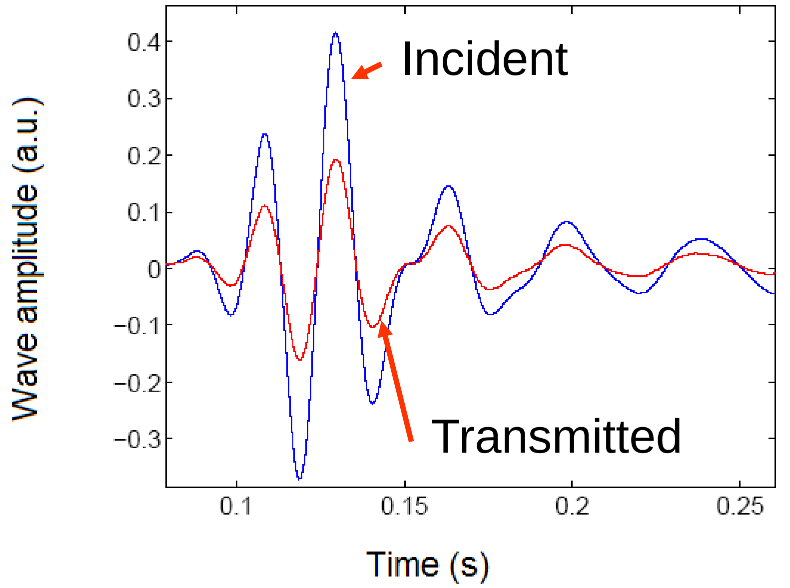


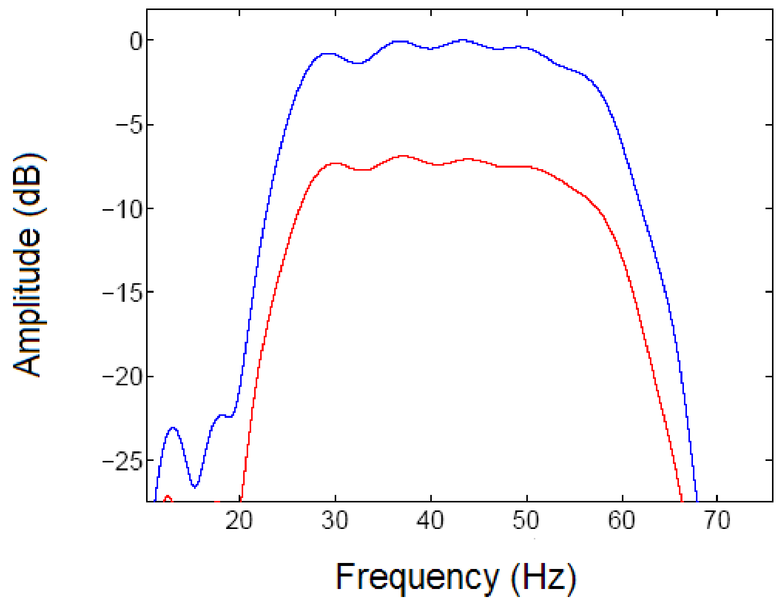
Plastic foams

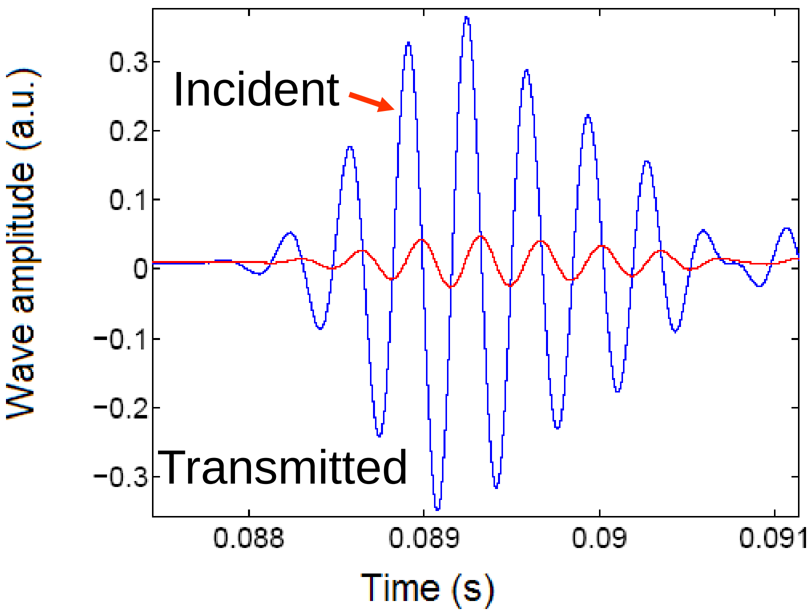


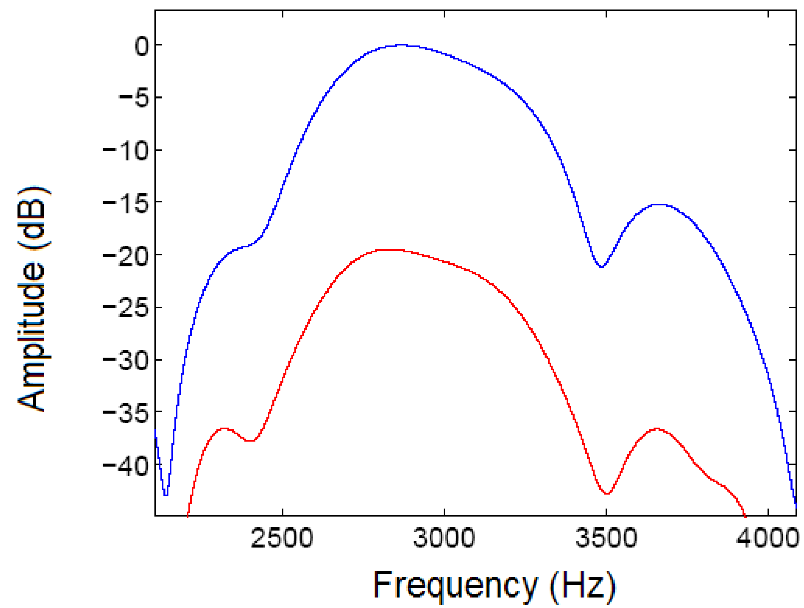
Experimental set-up of low frequency measurements (20 Hz – 3 KHz).




Incident
Transmitted
Wave amplitude (a.u.)
Time (s)
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
0.1
0.15
0.2
0.25


Amplitude (dB)
Frequency (Hz)
0
−5
−10
−15
−20
−25
20
30
40
50
60
70


Incident
Transmitted
Wave amplitude (a.u.)
Time (s)
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
0.088
0.089
0.09
0.091


Amplitude (dB)
Frequency (Hz)
0
−5
−10
−15
−20
−25
−30
−35
−40
2500
3000
3500
4000

Equivalent fluid model

Free fluid

$$\rho_f \frac{\partial \mathrm{v}}{\partial \mathrm{t}} = -\nabla p,$$

$$K_a^{-1} \frac{\partial \mathrm{p}}{\partial \mathrm{t}} = -\nabla \mathrm{v},$$

porous material

$$\rho(\omega) \frac{\partial \mathrm{v}}{\partial \mathrm{t}} = -\nabla p,$$

$$K^{-1}(\omega) \frac{\partial \mathrm{p}}{\partial \mathrm{t}} = -\nabla \mathrm{v},$$

$$\rho(\omega) = \rho_f \alpha(\omega) \qquad K^{-1}(\omega) = K_a^{-1} \beta(\omega)$$

$\alpha(\omega)$: *dynamic tortuosity*

$\beta(\omega)$: *dynamic compressibility*

Very low frequency range

$$\delta = \sqrt{\frac{2\eta}{\omega\rho_f}} \gg \text{radius of pores}$$

Viscous skin thickness

$$\alpha(\omega) = \frac{\eta\phi}{j\omega\rho_f k_0}$$

$$\beta(\omega) = \gamma,$$

$$\frac{\partial^2 p(x,t)}{\partial x^2} - D\frac{\partial p(x,t)}{\partial t} = 0$$

Diffusion equation (Darcy's law)

$$D = \frac{\eta\phi\gamma}{k_0 K_a}$$

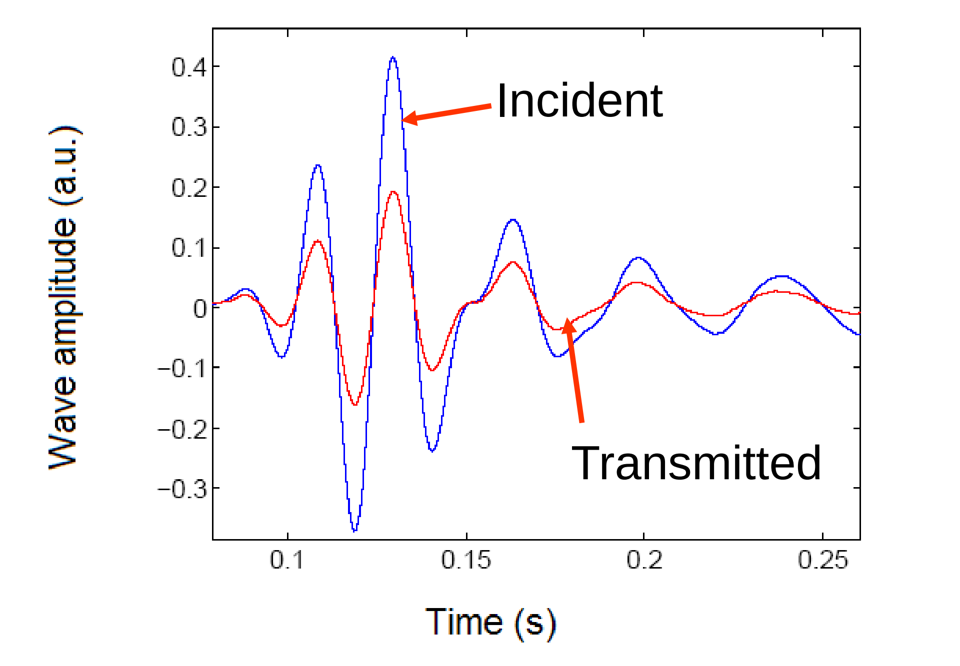
k_0 : Viscous permeability

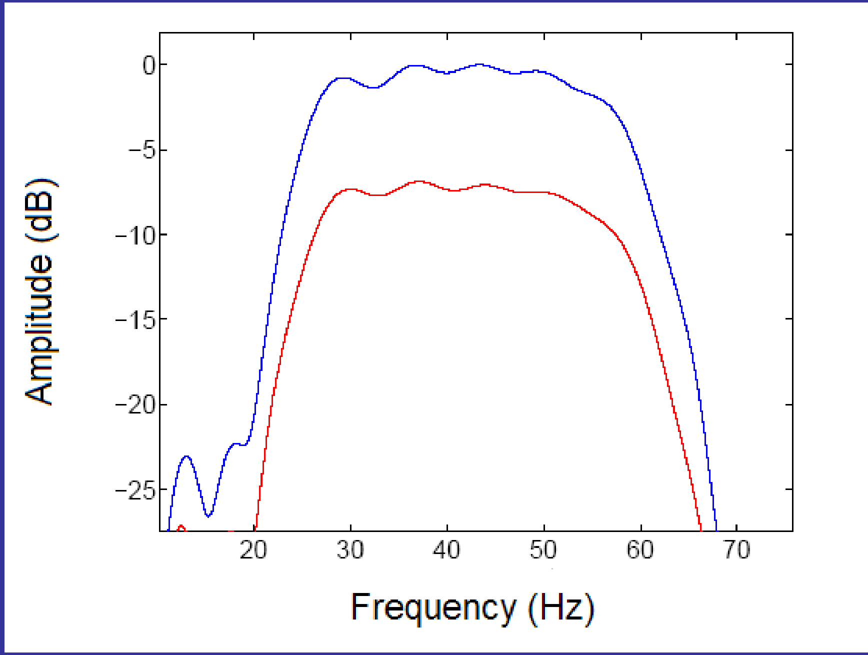
$$k_0 = \frac{\eta}{\sigma}$$

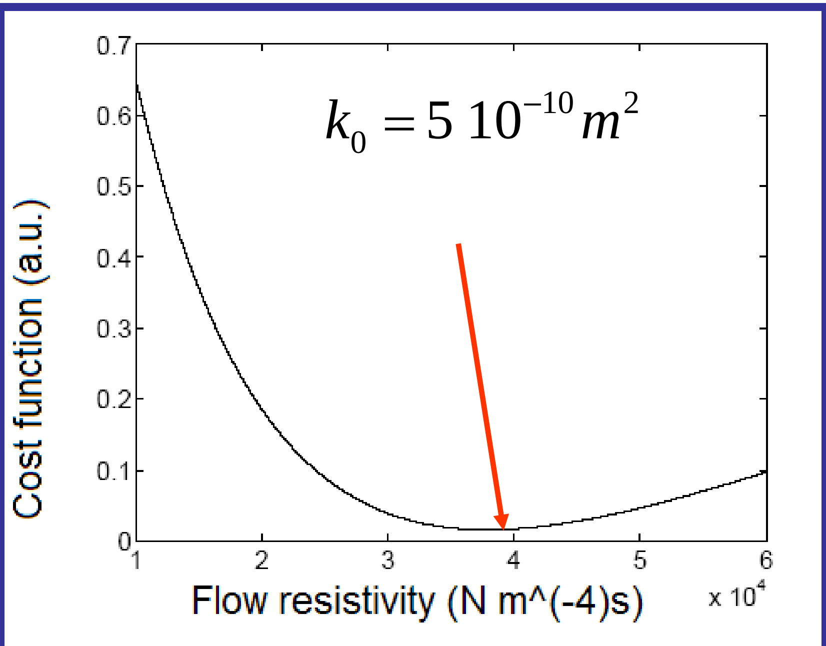
Fluid viscosity (η)

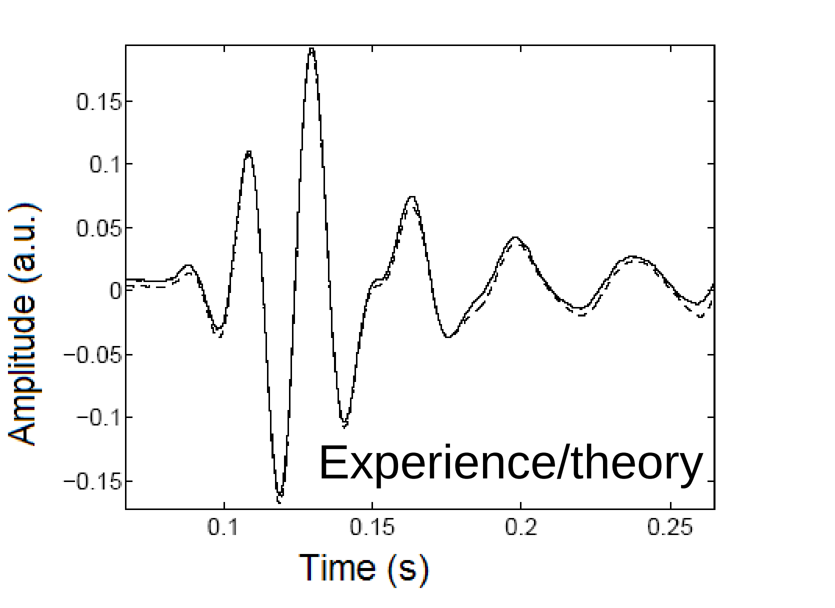
Flow resistivity (σ)

Measuring viscous permeability









Low frequency range

$$\delta = \sqrt{\frac{2\eta}{\omega\rho_f}} \gg \text{radius of pores}$$ ← Viscous skin thickness

$$\alpha(\omega) = -\frac{\eta\phi}{j\omega\rho_f k_0} + \alpha_0 + \frac{2\alpha_\infty^4 k_0^3 \rho}{\eta\Lambda^4\phi^3 p^3} j\omega + \ldots$$

$$\beta(\omega) = \gamma + \frac{(\gamma-1)k_0^{'} P_r \rho}{\eta\phi} j\omega - \frac{\alpha_0^{'}(\gamma-1)k_0^{'2} P_r^2 \rho^2}{\eta^2\phi^2}\omega^2 + \ldots$$

α_0 : Viscous tortuosity

$k_0^{'}$: Thermal permeability

$\alpha_0^{'}$: Thermal tortuosity

Propagation equation

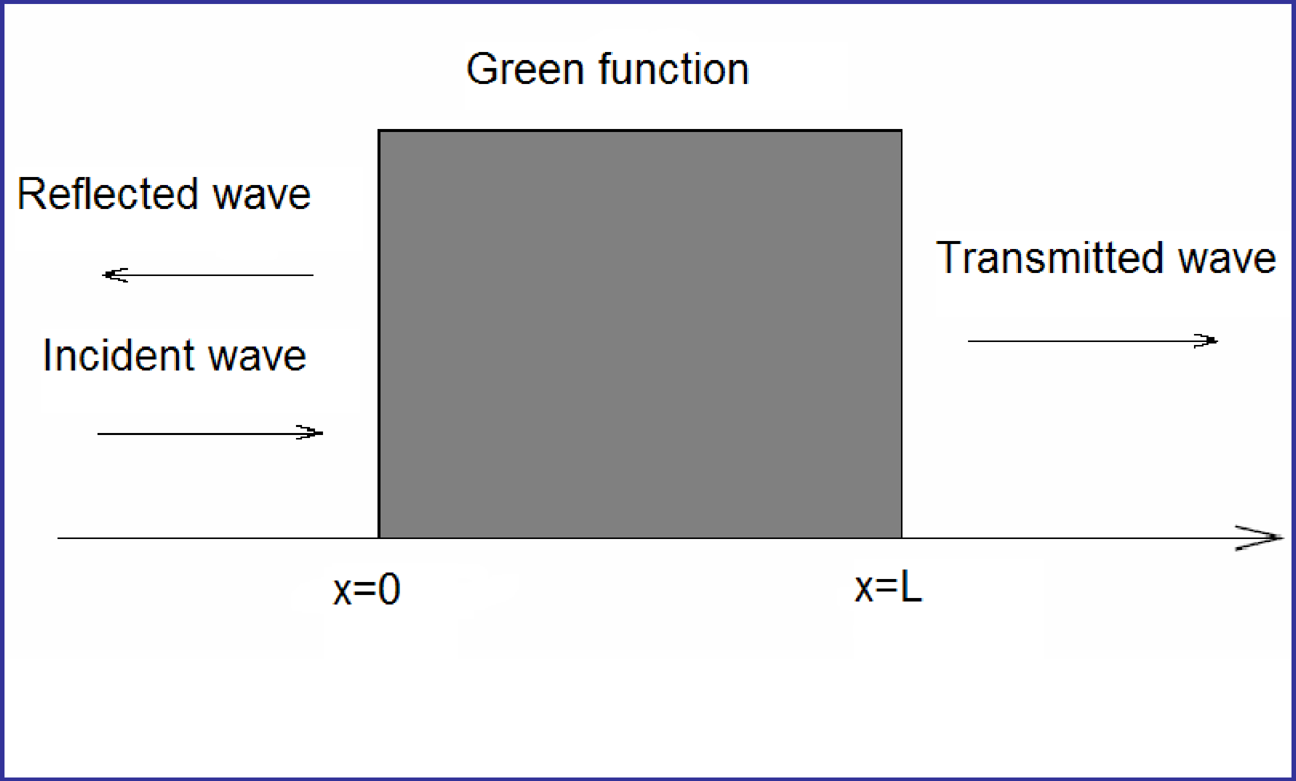
$$\frac{\partial^2 p(x,t)}{\partial x^2} - D\frac{\partial p(x,t)}{\partial t} - \frac{1}{c^2}\frac{\partial^2 p(x,t)}{\partial t^2} + H\frac{\partial^3 p(x,t)}{\partial t^3} = 0,$$

$$\frac{1}{c^2} = \frac{\rho_f}{K_a}\left(\alpha_0\gamma - \frac{(\gamma-1)P_r k_0^{'}}{k_0}\right),$$

$$D = \frac{\eta\phi\gamma}{k_0 K_a}$$

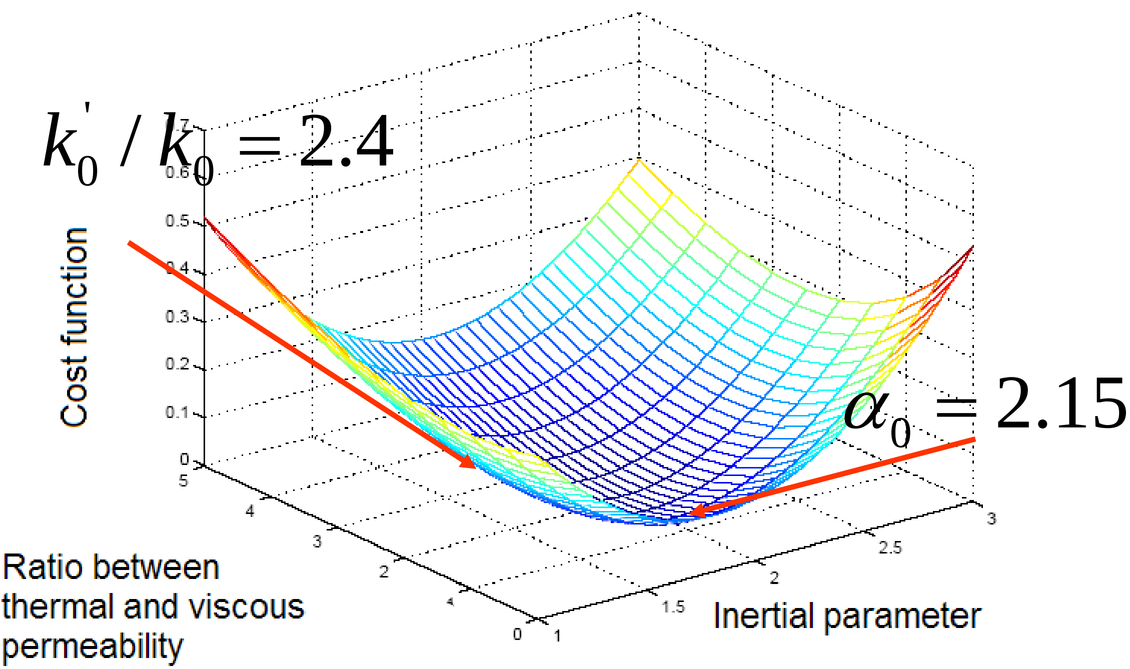
$$H = \frac{\alpha_0(\gamma-1)k_0^{'}P_r\rho^2}{K_a\eta\phi} - \frac{\alpha_0^{'}k_0^{'2}P_r^2\rho^2}{\eta\phi k_0 K_a} + \frac{2\alpha_\infty^4 k_0^3\rho^2\gamma}{K_a\eta\Lambda^4\phi^3 p^3}$$

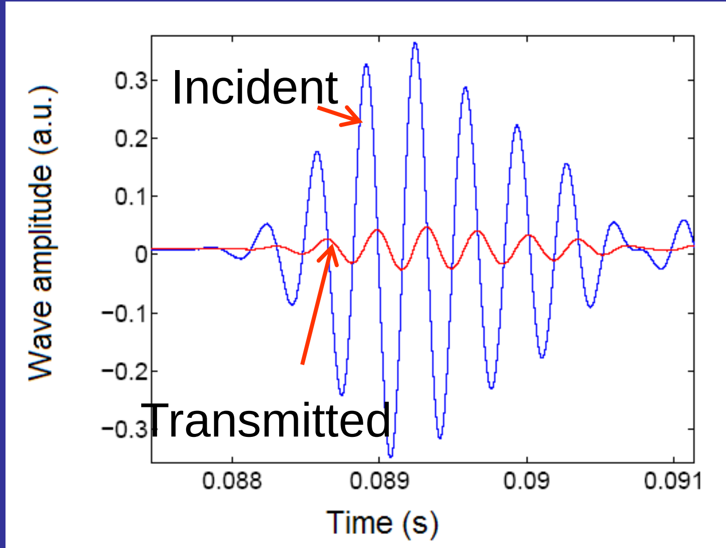
Direct problem, Inverse problem

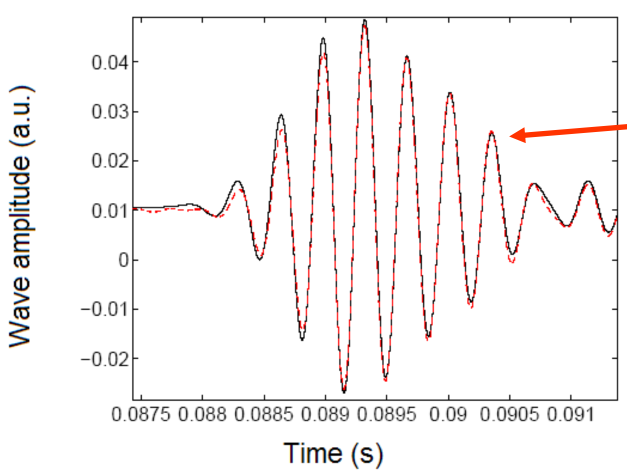


Cost function: $U\left(\alpha_0, k_0^{'}\right) = \sum_{i=1}^{i=N} (p_i^{\exp} - p_i^{th})^2$

Measuring thermal permeability $k_0^{'}$ and the viscous tortuosity α_0



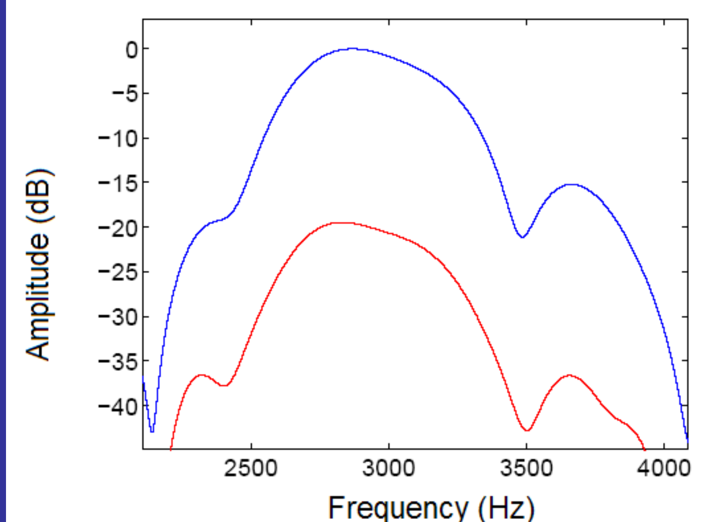




Experiment/theory

$$\alpha_0^{'} = \frac{\alpha_0}{\alpha_\infty}$$

$$\phi = 0.9$$



Conclusions and perspectives

- Experimental determination of the viscous tortuosity α_0 and the thermal permeability $k_0^{'}$ by solving the inverse problem.
- Simple, rapid and efficient method for non resistive porous materials.
- Biot's vibrations prevent applying this method for resistive porous materials.
- Measuring the thermal tortuosity $\alpha_0^{'}$ using reflected and transmitted waves.

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- Questions contact: Fellah@lma.cnrs-mrs.fr

- References

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- Z.E.A. Fellah, M. Fellah, F.G. Mitri, N. Sebaa, W. Lauriks and C. Depollier, J. Appl. Phys. (102), 084906-6, 2007.