HOMOGENIZATION OF WAVE PROPAGATION IN PERIODIC FLUID SATURATED POROUS MEDIA

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INTRODUCTION

The paper deals with modeling wave propagation in the fluid-saturated porous medium (FSPM) with periodic structure at the mesoscopic scale, where individual fluid-filled pores are not distinguishable. At any point of the bulk material both the solid and fluid phases are present according to the volume fractions. In particular, we consider the Biot compressible model. The dynamic problem formulation involving displacements $\boldsymbol{u}$, seepage velocity $\boldsymbol{w}$ and pressure p is constituted by the momentum equation $(1)_1$, by the Darcy law $(1)_2$, relating the total fluid pressure to the seepage velocity, and by the fluid volume conservation $(1)_3$,

$$
\begin{aligned}
-\nabla\cdot(\boldsymbol{I\!D}\boldsymbol{\epsilon}(\boldsymbol{u}))+\nabla(\boldsymbol{\alpha}p)+\overline{\rho}\ddot{\boldsymbol{u}}+\rho^f\dot{\boldsymbol{w}}&=\boldsymbol{f}\ ,\\
\rho^f\ddot{\boldsymbol{u}}+\rho^w\dot{\boldsymbol{w}}+\boldsymbol{K}^{-1}\boldsymbol{w}+\nabla p&=0\\
\boldsymbol{\alpha}:\boldsymbol{\epsilon}(\dot{\boldsymbol{u}})+\mathrm{div}\boldsymbol{w}+\frac{1}{\mu}\dot{p}&=0\ .
\end{aligned}
\tag{1}
$$

where $\boldsymbol{\epsilon}(\boldsymbol{u})$ is the strain, ρ are densities, $\boldsymbol{K}$ is the permeability and coefficients $\boldsymbol{I\!D},\boldsymbol{\alpha},\mu$ determine the poroelasticity properties.

HOMOGENIZED MODEL

Assuming an ε-periodic structure generated by the representative microscopic cell $Y=]0,1[^3$, such that all coefficients of (1) are defined as periodic functions of $y=x/\varepsilon\in Y$, we apply the two-scale convergence method to analyze sequences of solutions $\{\boldsymbol{u}^\varepsilon,\boldsymbol{w}^\varepsilon,p^\varepsilon\}_\varepsilon$ for $\varepsilon\to 0$. The limit system of equations describing the dynamic response of the homogenized FSPM can be presented in the operator form for the Laplace-transformed time domain (below λ is the Laplace variable and $\mathcal{L}:f(t)\mapsto\mathcal{L}\{f\}(\lambda)$, ∇^S is the symmetric gradient)

$$
\left[\lambda^2\mathbb{M}_{\mathcal{L}}(\lambda)+\lambda\mathbb{D}_{\mathcal{L}}(\lambda)+\mathbb{G}+\mathbb{K}\right]\boldsymbol{q}_{\mathcal{L}}=\boldsymbol{f}_{\mathcal{L}}\ ,
$$

where

$$
\mathbb{M}_{\mathcal{L}}(\lambda)=\begin{pmatrix}\boldsymbol{\mathcal{M}}(\lambda) & 0\\ 0 & 0\end{pmatrix}\qquad
\mathbb{K}=\begin{pmatrix}-\nabla^S\cdot(\boldsymbol{I\!D}:\nabla^S\circ) & 0\\ 0 & \mathcal{Q}\end{pmatrix},
$$

$$
\mathbb{D}_{\mathcal{L}}(\lambda)=\begin{pmatrix}0 & -\rho^f\boldsymbol{\mathcal{K}}(\lambda)\cdot\nabla\circ\\ -\nabla\cdot(\rho^f\boldsymbol{\mathcal{K}}(\lambda)\cdot\circ) & -\lambda^{-1}\nabla\cdot(\boldsymbol{\mathcal{K}}(\lambda)\cdot\nabla\circ)\end{pmatrix}\qquad
\mathbb{G}=\begin{pmatrix}0 & \nabla^S\cdot(\mathcal{A}\cdot\circ)\\ \mathcal{A}:\nabla^S\circ & 0\end{pmatrix}
$$

$$
\boldsymbol{q}_{\mathcal{L}}(\lambda)=\begin{pmatrix}\mathcal{L}\{\boldsymbol{u}\}\\ \mathcal{L}\{p\}\end{pmatrix},\qquad
\boldsymbol{f}_{\mathcal{L}}=\begin{pmatrix}\mathcal{L}\{\boldsymbol{f}\}\\ 0\end{pmatrix}.
$$

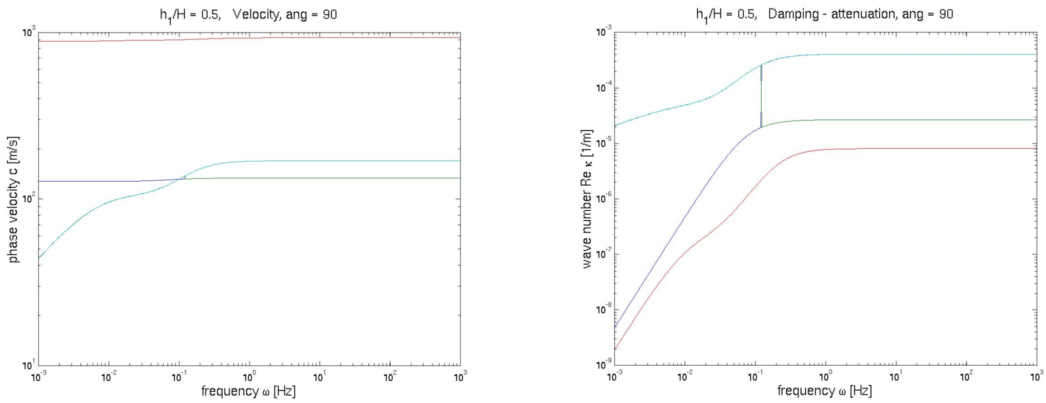

Figure 1. Dispersion curves for a laminated medium: phase velocity (left) and attenuation (right) w.r.t. frequency. Incidence angle 90 deg. w.r.t. the lamination.

The stationary homogenized coefficients $\boldsymbol{\mathcal{D}}, \mathcal{A}$ and $\mathcal{Q}$ are evaluated upon solving stationary microscopic poro-elasticity problems defined in cell Y. Homogenized mass $\mathcal{M}_{ij}(\lambda)$ and permeability $\mathcal{K}_{ij}(\lambda)$ coefficients depend on the local responses $\eta^k(y,\lambda)$ periodic in $y \in Y$ and satisfying

$$\mathrm{div}_y \left[\boldsymbol{F}(\lambda) \cdot \nabla_y (\mathcal{L}\{\eta^k\} - \frac{1}{\lambda} y_k) \right] = 0 \quad \text{a.e. in } Y \;, \quad k = 1,2,3 \;,$$

where $\boldsymbol{F}(\lambda) = [\lambda \phi_0^{-1} \rho^f \boldsymbol{I} + \boldsymbol{K}^{-1}]^{-1}$ is the ("frequency-dependent") permeability.

RESULTS

Using the homogenized model we study dispersion of the periodic medium assuming incidence of plane harmonic waves. For layered (laminated) medium we derive analytic expressions for computing the homogenized coefficients and obtain biquadratic equations for evaluating the dispersion of P and S waves. Upon analyzing asymptotic behaviour of the homogenized mass $\mathcal{M}_{ij} = \bar{\rho}_Y - \mathrm{i}\omega(\rho^f)^2 \mathcal{K}_{ij}(\mathrm{i}\omega)$ with respect to frequency ω we observed that for $\omega \to \infty$ only the solid phase mass induces the inertia effects and no damping is produced by $\mathcal{M}_{ij}(\mathrm{i}\omega)$, while for $\omega \to 0$ both the fluid and the solid masses participate according to the volume fractions and some damping is produced by the homogenized permeability. Using numerical methods an arbitrary microstructure can be studied. Further we focus on waves in double porosity FSPM.

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