An efficient Wave Based Method to predict the response of multilayer systems containing poroelastic materials

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Wave Based Method, Biot theory, multilayer, Trefftz-approach

EXTENDED ABSTRACT

The Wave Based Method (WBM) [1] is a deterministic prediction technique which is based on an indirect Trefftz approach. Contrarily to element-based approaches it partitions the problem domain into a small number of large subdomains. Convexity of the subdomains is a sufficient condition for the method to converge towards the exact solution of the problem. Within each subdomain the dynamic response variables are approximated by a function series expansion of wave functions which are exact solutions of the governing differential equations. As compared to element-based approaches, the resulting numerical models are much smaller, enabling solutions at higher frequencies. The method has already been successfully applied for 2D and 3D (vibro-) acoustic problems, both bounded and unbounded, and structural problems

Recently, the method has also been applied to deal with the Biot equations [2] for poroelastic materials [3]. To be used within the WBM, the Biot equations are decoupled into three Helmholtz equations by means of a strain or a potential formulation. Three sets of wave functions are then applied in each subdomain, capturing the physics of the three wave types which according to the Biot theory exist in poroelastic materials. The wave functions are solutions to the governing differential equations and thus only violate the boundary and interface conditions. These errors on the boundaries and interfaces are minimised using a Galerkin weighted residual approach. The resulting system of equations can be solved for the unknown contribution factors of each wave function. The associated number of degrees of freedom is again small, enabling an efficient solution up to higher frequencies.

Generally, the Finite Element Method (FEM) takes advantage of sparse and frequency-independent matrices which may be solved in an efficient manner. However, concerning poroelastic materials, the matrices become complex and frequency-dependent due to the material properties, hampering the effectiveness of the method. When coupling acoustic or elastic problem domains together with poroelastic element descriptions, the nice system matrix properties are countered because of these disadvantageous properties of poroelastic FE models. Also, very fine meshes are needed to capture the localised near-field effects that can exist at interfaces.

Since in the WB approach, the matrices are always frequency-dependent and complex, adding a poroelastic layer does not change the system properties drastically. This can be an advantage when studying multilayer systems. Also, since the WBM is a deterministic method and leads to exact solutions of the considered problem, finite sizes are automatically taken into account. Moreover, the layers do not necessarily need to be flat, as is the case for the Transfer Matrix Method (TMM) and the finite TMM (FTMM). This work evaluates the potential of the WBM to deal with different combinations of poroelastic and acoustic layers, with varying thicknesses, as compared to the FEM.

A first example shows a multilayer, consisting of 3 poroelastic layers and 2 different materials as shown in figure 1. The top layer is excited with a distributed acoustic pressure, $p=2x^3-3x^2+1$ Pa

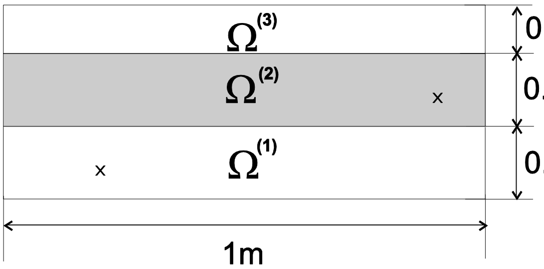

Figure 1: Problem definition

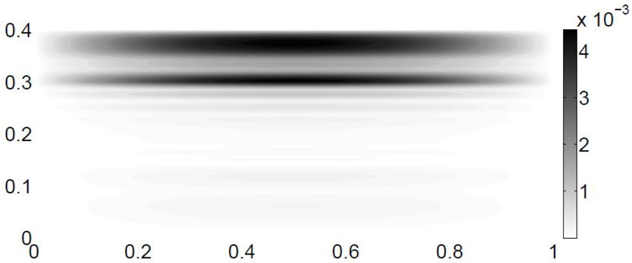

Figure 2: shear stress field at 600Hz

Figure 2 shows the calculated shear stress field within the material, calculated at 600Hz, indicating this field is continuous over the different subdomains. The convergence curves below show the relative averaged error on the displacement field of the fluid phase at two frequencies as compared to the finest FE model. Note that the stagnation of the WBM convergence curve is below the error of the finest FE model and is expected to be due to the accuracy of the FE reference. A very good accuracy is obtained with the WBM in a short calculation time as compared to cubic adaptively refined FE results indicating the usability of the WBM to model different materials together.

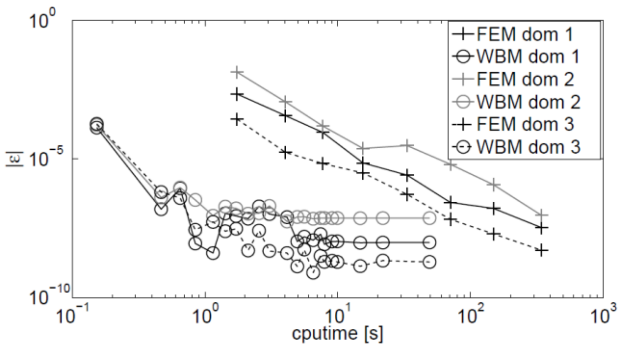

Figure 3: convergence at 200Hz

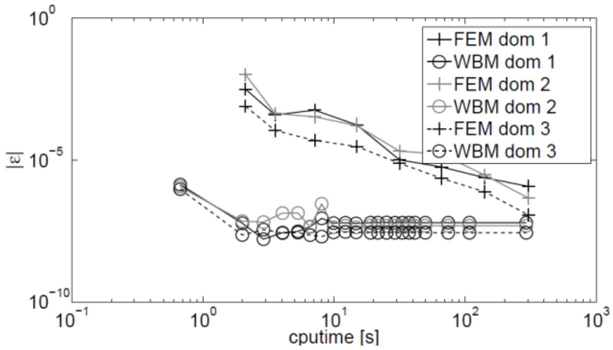

Figure 4: convergence at 400Hz

The presentation explains in detail the theory, the mathematical background and various examples containing air layers and poroelastic layers. Also the effect on accuracy is discussed when the different layers become more slender. The presentation gives the drawbacks and limitations of this method and gives an overview of futures research topics which come into reach.

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