On cylindrical waves in anisotropic Cartesian materials and its implications on the harmonic mode separability assumption

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The objective here, is to investigate wave propagation in rotationally symmetric geometries consisting of anisotropic media where the principal directions of the Hookes tensor is not aligned with the cylindrical coordinate system. This will result in spatially varying material parameters, influencing the assumption of harmonic, orthogonal, modes with respect to the azimuthal angle.

When studying wave propagation in rotationally symmetric structures consisting of (visco-)elastic and poro-elastic media, the wave fields may be separated by means of a Fourier expansion for the azimuthal angle φ, that is, a load described by a traction $\mathbf{T}(r,\varphi,z) = \mathbf{t}(r,z) \cdot \exp(\mathrm{i}m\varphi)$ will result in a response, here given as displacement, as $\mathbf{U}(r,\varphi,z) = \mathbf{u}(r,z) \cdot \exp(\mathrm{i}m\varphi)$, employing the orthogonal properties of the Fourier series. This is approach is valid for both isotropic and anisotropic media; a restriction being that the Hookes tensor must be constant in the assigned cylindrical coordinate system. However, considering a material where the principal directions coincide with a Cartesian coordinate system, the Hookes tensor will be spatially varying in cylindrical coordinates and may similarly be described by a Fourier series expansion as

$$\mathbb{C} = \sum_{m=-4}^{4} \mathbb{C}_m \mathrm{e}^{\mathrm{i}m\varphi}, \tag{1}$$

where the nine (inter-related) components $\mathbb{C}_m$ are sufficient to fully describe the elastic properties. This will cause the de-coupling of the circumferential modes to break down. For example, an azimuthally constant (zero order) load, $\mathbf{T}(r,\varphi,z) = \mathbf{T}(r,z)$, will cause a response consisting of a series of displacement components,

$$\mathbf{U} = \sum_{m=-\infty}^{\infty} \mathbf{u}_m \mathrm{e}^{\mathrm{i}m\varphi} \tag{2}$$

and hence spreading the modes to a certain degree. The distribution of these modes depend on the material properties; the kind of anisotropy will decide to what extent the response will spread to higher order modes. From a vibration attenuation perspective, a high degree of such mode spreading would arguably be of interest; in the extreme, however un-physical, case, the energy would be evenly distributed over an infinite number of modes, hence causing the energy in each mode to approach zero. In particular, the cases of transversely isotropic, orthotropic and fully anistropic materials will be discussed and possible implications in terms of mode spreading of these will be highlighted.