A hybrid Finite Element – Wave Based Method for coupled acoustic-poroelastic applications

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EXTENDED ABSTRACT

To assess and optimise the effect of stacked multi-layers which are used for reduction of sound and vibration transmission, design engineers greatly depend on CAE-tools. Mostly these are based on the Finite Element Method (FEM) or the Boundary Element Method (BEM). Both of these methods are, however, in practice limited to low-frequency simulations due to the strongly increasing computational cost with the frequency.

These classical methods lose even more of their applicability when used for poroelastic [1][2] materials due to a number of supplementary phenomena. Firstly, the high number of degrees of freedom per node, depending on the used formulation (6 for (u-U) [3], 4 for (u-p) [4]) increases the bandwidth of the matrices. Secondly, the operating principle of poroelastic materials is based on localised near-field effects, which require a very fine element discretisation, not only in the poroelastic material itself, but also in the surrounding (vibro-) acoustic system. Thirdly, the (strongly) frequency dependent material parameters cause the system matrices to be frequency dependent, thus reducing or even impeding the applicability of modal reduction schemes.

With the current state of the art, it is very challenging to solve a full FE-model for the poroelastic material into the mid-frequency domain, let alone a fully coupled (vibro-) acoustic model incorporating these materials. To ease this large increase in solving time when going to full-system modelling, other approaches than element based methods can be considered. The Wave Based Method (WBM) [5] is very well suited for this full-system modelling, since it has a very nice convergence rate as compared to the FEM and BEM. It has already been applied to a large number of problem types: acoustic, structural (both plate membrane and bending behaviour) and poroelastic problems. The method uses an indirect Trefftz-approach; the field variables are written as a weighted sum of so-called wave functions and particular solutions, both of which satisfy the governing differential equations. The boundary conditions and coupling conditions are then approximated in a weighted residual sense. This leads to a small, fully populated, complex and frequency dependent system in the unknown wave function contributions.

The application of the WBM is, however, limited to convex domains and is thus only applicable if the geometry can be divided into a (small) number geometrically simple, convex subdomains. To overcome this limitation, a hybrid Finite Element - Wave Based method has been developed for different sorts of hybrid coupling types: acoustic [6], structural [7] and structural-acoustic [8] couplings.

This present contribution continues the work on these hybrid FE-WB methods towards coupling WB models for acoustics with FE models for poroelastics, both for a (u,p) and a (u,U) formulation. In this way, the method benefits from the computational efficiency of the Wave Based Method for the acoustic calculations, without losing the Finite Element Method's ability to model the often layered and complexly shaped poroelastic material in great detail.

The presentation focuses on the mathematical formulation and methodological side of this hybrid coupling. The benefits and disadvantages of two different hybrid approaches, one using the (u,p) formulation in the FE subdomain and the other using the (u,U) formulation, are discussed and their differences are explained. The preferred method proves to be dependent on the relative size of the poroelastic domain to the acoustic domain. When this ratio is large, the (u,p) formulation is preferred due to the smaller FE models because of the lower number of degrees of freedom (4 instead of 6). As the ratio decreases, which is the case where thin (multi-) layers are used in larger acoustic cavities, the (u,U) formulation benefits from the fact that the coupling to the Wave Based acoustic domain is more natural, contrarily to pure FEM where the (u,p) couples best.

The presented methods are validated with a numerical example, illustrating the potential for reaching into the mid-frequency range. This study considers the interior acoustics of a rigid-walled cavity where one of the walls is covered with a layer of damping foam.

The development of these novel hybrid couplings between the acoustic WBM and the poroelastic FEM is an important step towards full system modelling of vibro-acoustic problems, where poroelastic materials are more and more used to meet noise control requirements. By combining the computational efficiency of the Wave Based Method for simply shaped acoustic cavities with the geometrical flexibility of the Finite Element Method for damping multi-layers of a complex shape, full-system simulations into the mid-frequency range come within reach.

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