Two-sample method for mechanical characterization of isotropic acoustic foams

S. Sahraoui[(*)], A. Geslain, B. Brouard
LAUM, CNRS UMR 6613, Université du Maine, Av. O. Messiaen, 72085 Le Mans, France
D. Parmentier
CTTM, 20 rue Thalès de Milet, 72000, Le Mans.

This paper presents a quasi-static method for the characterization of the viscoelastic properties of polyurethane foams. A compression test setup is used to measure the compression stiffness of cubic sample for checking the isotropy or for evaluating the degree of anisotropy. For isotropic materials the Young's modulus and the Poisson's ratio are determined through normalised stiffness ratios built by the finite element method for brick-shaped at various shape factors. The measured stiffness of samples cut in a foam plate and the juxtaposition of two samples give an easy extraction of the elastic constants at low frequency. This approach is extended to disk-shaped samples.

1. Introduction

Acoustic propagation in the open cell foams can be predicted with a phenomenological model of the Biot theory[1.2] where the elastic properties of the solid phase are needed. To characterize the viscoelastic properties of acoustic foams, only a few experimental techniques are available in the literature and they consist on non resonant techniques,[3-5] standing wave resonance of a longitudinally excited rod with end mass[6,7] or mass-spring resonance.[8] They all perform measurements on a narrow frequency range. An extension of those measurements on a wide frequency range has been made by theoretical modelization of the viscoelastic behavior of foams,[9] through the frequency-temperature superposition principle[10] or by means of acoustical excitations.[11,12]

Due to the manufacturing process, porous materials, such as foamed polymers, exhibit elastic properties that are highly anisotropic. In the rising direction, non-regular cell geometry is induced which, in general, leads to highly directive cell orientations, with strength and stiffness found to be higher in the direction of the elongated cells.[13-15] In these studies, cubic sample were considered to investigate the mechanical response of each direction; Mariez et al.[15] and Melon et al.[14] check the axisymmetrical behavior of the sample and more recently, Guastavino et al.[13,16] give a mapping of the 3D full-field displacement and show complex strain on the four free faces due to foam anisotropy and the boundary conditions; a methodology for identification of general orthotropic elastic models of porous foams is proposed. The collapse of the cells is proceeding following a process of static compression and the influence of the no-slip boundary conditions pointed out in his thesis are described in a forthcoming paper.[17]

In the first work by Mariez et al., a cubic foam sample is compressed between two rigid plates - see Fig. 1. The Young modulus E and the Poisson's ratio υ are adjusted in a static finite element model of the test setup to match the measured mechanical impedance (*F/v*) and the displacement ratio. This method requires the measure of the lateral displacement of the face centre by a laser vibrometer. In the second work by Langlois et al. two cylindrical samples of different shape factors are compressed with the same setup and then the two elastic constants on the material are determined from the impedance measurements and the polynomial relations derived from finite element simulations.

(*) Corresponding author: Sohbi.Sahraoui@univ-lemans.fr

This paper is focalised on the two-samples method proposed by Langlois et al.[5] with cylindrical samples and extended here to parallelepipedic geometry and where a better alternative is found without using the polynomial relations. The investigation on cubic sample let us to check the isotropy on the foam or at less the degree of anisotropy. The introduction of normalised stiffness ratio calculated by finite element simulation give a straightforward access to E and υ. The calculations were performed for various shape factors and Poisson's ratio (0 to 0.48 by 0.005 step). Here the shape factor is defined as the side of the square section (or diameter of circular section)-to-the thickness ratio.

2. Finite element simulation of static compression

2.1 Parallelepipedic samples

A foam sample with the Young's modulus E and the Poisson's ratio υ has the stiffness:

$$K = F / v \quad (1)$$

where F and v are respectively the compressive load measured by the load sensor and the vertical displacement measured by the inductive sensor as indicated in Fig. 1a.

For parallelepipedic samples having a square section ($A=L^2$) the thickness value is given by L/s where s is the corresponding shape factor. The stiffness K [Eq.(1)], depending on the elastic constant and the sample dimensions, can be written as:

$$K_s = \frac{EA}{L/s} k_s(\upsilon) \quad (2)$$

where K_s and k_s are respectively the measured and normalised stiffness functions.

For a half parallelepipedic sample, the thickness and the shape factor are respectively h/2 and 2s, the stiffness is:

$$K_{2s} = \frac{EA}{L/2s} k_{2s}(\upsilon) \quad (3)$$

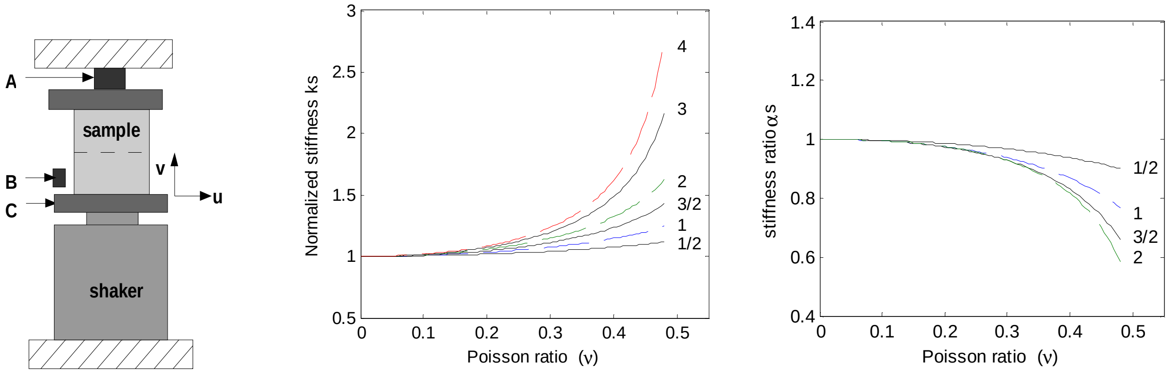
where K_{2s} and k_{2s} are respectively the measured and normalised stiffness functions.

By using Eqs (2) and (3) the normalised stiffness ratio α_s takes the form:

$$\alpha_S(\upsilon) = \frac{k_s}{k_{2s}} = \frac{2K_s}{K_{2s}} \quad (4)$$

where the ratio K_s / K_{2s} is only depending on Poisson's ratio. The normalised stiffness k_s and the ratio $\alpha_s(\upsilon)$ are plotted in Fig. 1b and 1c. We can observe on these simulated curves that k_S are equal to 1 for low Poisson's ratio; in this case the simulation confirms that the sample doesn't bulge sideways when lateral deformation is not allowed in the loaded faces.
The bijective function $\alpha_s(\upsilon)$ plotted in Fig. 1c gives a straightforward access to E and υ; Poisson's ratio is firstly obtained from the measured stiffness of each sample [Eq. (4)]. Secondly, Young's modulus is determined from Eq. (2) as:

$$E = \frac{K_s}{sLk_s(\upsilon)} \quad (5)$$



a) b) c)

FIG. 1: Compression test
a) Experimental setup (A: load F sensor, B: displacement v sensor of the moving plate C)
b) Numerical stiffness vs Poisson's ratio for various shape ratios
c) Normalized stiffness ratio

2.2. Case of cubic samples

For a cubic sample (s=1) and a half cube (s=2), the stiffness is:

$$K_1 = \frac{EA}{L} k_1(\upsilon) \tag{6}$$

and:

$$K_2 = \frac{EA}{L/2} k_2(\upsilon) \tag{7}$$

where the ratio K_1 / K_2 is only depending on Poisson's ratio. The normalised stiffness ratio α_1 takes the form:

$$\frac{k_1(\upsilon)}{k_2(\upsilon)} = \frac{2K_1}{K_2} = \alpha_1(\upsilon) \tag{8}$$

The equations (6-8) can be obtained from the general equations (2-4).

2.3. Cylindrical samples

For cylindrical samples having a circular section ($A=\pi D^2/4$) the thickness value is given by D/s. Then Eq. 2 takes the same form:

$$K_s = \frac{EA}{D/s} k'_s(\upsilon) \tag{9}$$

Similarly we can build numerically new normalized stiffness k'_s and stiffness ratio as explained above for parallelepipedic samples. Therefore the experimental values of K_s and K_{2s} let us to calculate firstly υ and secondly E by using Eq. 6:

$$E = \frac{4K_s}{\pi s D k'_s(\upsilon)} \tag{10}$$

The normalized stiffness and the normalized stiffness ratio are calculated for the shape factor of sample used in the characterization section.

3. Measurements of Poisson's ratio and Young's modulus

We limit the stiffness measurements at one frequency (40 Hz) and the initial applied strain and the amplitude are fixed at $\varepsilon_0 = 3\%$ and $\varepsilon_1 = 0.1\%$ respectively by using the experimental setup schematised by figure 1a. Then, in complex notation, the applied strain is :

$$\hat{\varepsilon} = \varepsilon_0 + \varepsilon_1 e^{j\omega t} \quad (11)$$

and the complex Young's modulus takes the form :

$$\hat{E} = E(1 + j\eta) \quad (12)$$

where E and η represent the Young's modulus and the loss factor.

3.1. Isotropy checking

The open-cell foams exhibit a quasi-axisymmetrical behavior[14,16] and the rising direction during the foaming process, named longitudinal direction, corresponds to one of the principal axis.

As mentioned above the cubic geometry allows us to check the isotropy or the degree of anisotropy by comparing the stiffness in the three directions. The experiments were performed with three foams plate having the same thickness (40 mm). Cubic samples (40x40x40) of the three different foams are investigated and we note K_T, $K_{T'}$ and K_L the value of the stiffness in each direction (transverse and longitudinal); the table I gives relative values where K_T is fixed to 100.

The polyurethane (PU) foams "A" and "B" and the melamine "C" (table I) show the extreme cases of low and high anisotropy; the foam A is the closest material to isotropy but we have to keep in memory that there is a substantial estimate (12% difference).

3.2. Sample juxtaposition

Shear stress arises from the no-slip conditions at the loading surfaces as mentioned before. The absence of shear stress in the horizontal plane of symmetry of the cubic sample is the basic property suggesting to test together two juxtaposed samples. All tests on entire cubic samples and on two half-cube samples give similar results. Therefore the two-sample method quoted above[5] can be applied to identical sample cut in the same foam plate; the stiffness measurements are performed on two juxtaposed samples and on one sample.

K_T	$K_{T'}$	K_L	foam
100	105	112	"A"
100	104	125	"B"
100	105	144	"C"

Table I: Relative value stiffness in principal directions

	Square section	circular section
E(kPa)	292	311
υ	0.42	0.44
η	0.12	0.11

Table II: Mechanical properties of PU foam at 40 Hz

3.3. Characterization example

Brick-shaped and disk-shaped sample are cut a PU foam plate being 20 mm in thickness; for square (40x40) and circular (D = 44 mm) section the shape factors are respectively 2 and 2.2. The measurements of identical samples stiffness show some fluctuation (5-10%) due to the foam heterogeneity and then the results in table II gives the mean value for each geometry. The Young's modulus measured at 3% initial strain corresponds at his maximal value; the influence of initial strain on E is widely discussed in a further paper[17].

4. Conclusion

A quasi-static mechanical characterization of acoustic foam is investigated with the two-sample method. The method uses two identical brick-shaped or disk-shaped samples cut in commercial available plate foam. The originality lies with the development of a normalized stiffness ratio for extracting the Young's modulus and the Poisson's ratio. Contrary to previous method, this approach doesn't require a polynomial calculation and the cut of two different shape factor samples. The main result of this work is linked to the use of cubic and half-cubic samples for evaluating the degree of anisotropy and to estimate the isotropy approximation.

References

[1] M.A. Biot, *Acoustic, Elasticity and Thermodynamics of Porous Media- Twenty One Papers by M.A. Biot,* edited by I. Tolstoy (American Institute of Physics, Woodbury, NY, 1992).

[2] J.F. Allard, *Propagation of Sound in Porous Media: Modelling Sound Absorbing Materials,* (Elsevier Applied Science, London, 1993).

[3] E. Mariez, S. Sahraoui, and J.F. Allard, *"Elastic constants of polyurethane foam's skeleton for Biot model,"* Proceedings of Internoise 96, pp.951-954 (1996).

[4] S. Sahraoui, , E. Mariez, and M. Etchessahar, *Mechanical testing of polymeric foams at low frequency.* Polym. Test. **20**, 93-96, (2001).

[5] C. Langlois, R. Panneton and N. Atalla, *"Polynomial relations for quasi-static mechanical characterization of isotropic poroelastic materials,"* J. Acoust. Soc. Am. **110,** 3032-3040, (2001)

[6] T. Pritz, *"Dynamics Young's modulus and loss factor of plastic foams for impact sound isolation,"* J. Sound Vib. **178**, 315-322, (1994).

[7] A. Sfaoui, *"On the viscoelasticity of polyurethane foams,"* J. Acoust. Soc. Am. **97**, 1046-1052, (1995)

[8] T. Pritz, *"Non-linearity of frame dynamic characteristics of mineral and glass wool materials,"* J. Sound Vib. **136**, 263-274, (1990).

[9] T. Pritz, *"Analysis of four-parameter fractional derivative model of real solid materials,"* J. Sound Vib. **195**, 103-115, (1996).

[10] M. Etchessahar, S. Sahraoui, L. Benyahia and J.F. Tassin, *"Frequency dependence of elastic properties of acoustic foams,"* J. Acoust. Soc. Am. **117**, 1114-1121, (2005).

[11] J.F. Allard, M. Henry, L. Boeckx, P. Leclaire and W. Lauriks, "*Acoustical measurement of the shear modulus for thin porous layers,"* J. Acoust. Soc. Am. **117**, 1737-1743, (2005).

[12] V. Gareton, D. Lafarge and S. Sahraoui, *"The measurement of the shear modulus of a porous polymer layer with two microphones,"* Polym. Test. **28**, 508-510, (2009).

[13] R. Guastavino and P. Göransson, "*A 3D displacement measurement methodology for anisotropic porous cellular foam materials,"* Polym. Test. **26**, 711-719, (2007).

[14] M. Melon, E. Mariez, C. Ayrault and S. Sahraoui, *"Acoustical and mechanical characterization of anisotropic open-cell foams,"* J. Acoust. Soc. Am. **104**, 2622-2627, (1998).

[15] E. Mariez and S. Sahraoui, *"Measurement of mechanical anisotropic properties of acoustic foams for the Biot model,"* Proceedings of Internoise 97, pp.1683-1686 (1997).

[16] R. Guastavino, "Elastic and acoustic characterization of anisotropic porous materials", Ph.D. dissertation, KTH Engineering Science, Stockholm, (2008).

[17] A. Geslain, O. Dazel, J-P. Groby, S. Sahraoui, and W. Lauriks, *"Influence of static compression on mechanical parameters of acoustic foams"*, accepted for publication in J. Acoust. Soc. Am.