The Question of the Existence of Other Propagating Compressional Waves in Porous Media

BOSTON UNIVERSITY

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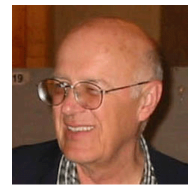
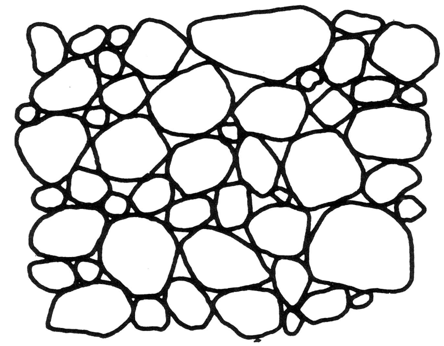




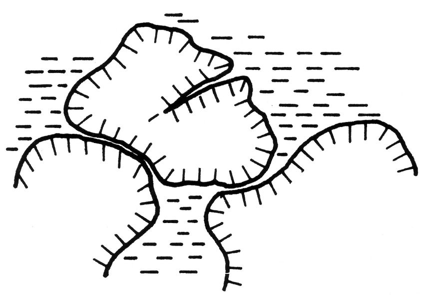
Symposium on the Acoustics of Poro-Elastic Materials

sapem 2011

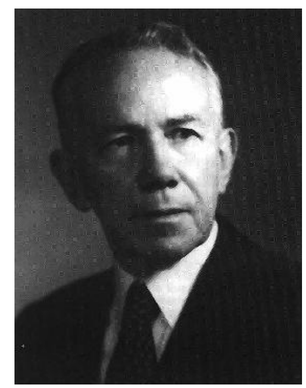
Ferrara, Italy

16 December 2011

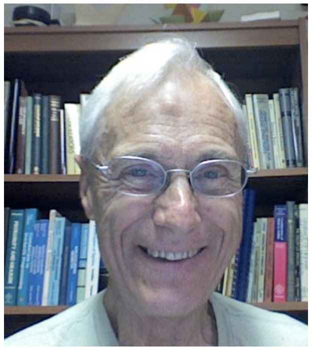
Yakov I. Frenkel

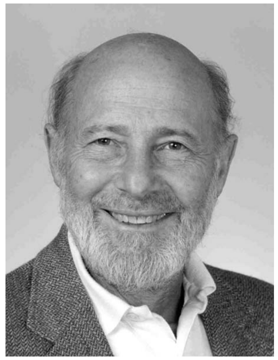


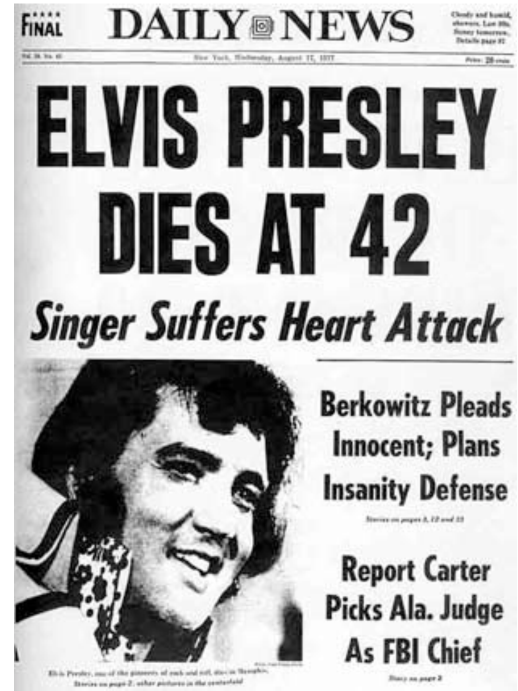
(Figure from Biot, JASA, 1962)



Maurice A. Biot







FINAL

DAILY NEWS

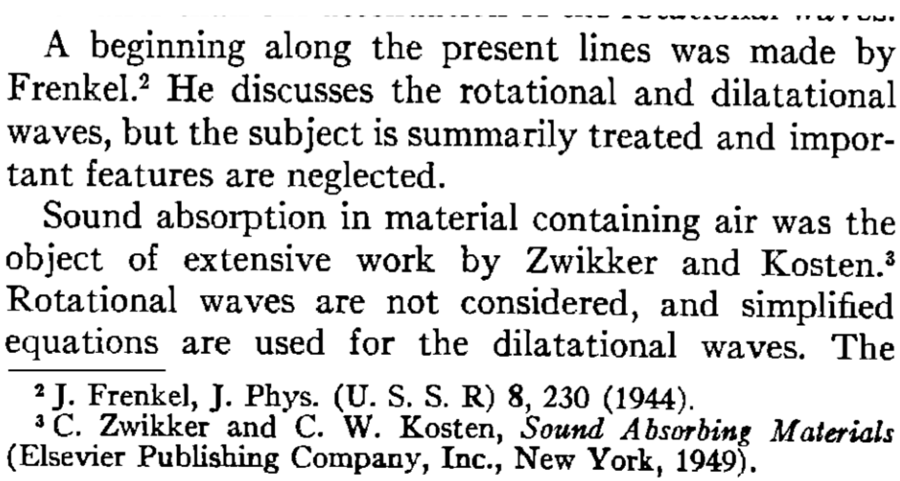
ELVIS PRESLEY DIES AT 42

Singer Suffers Heart Attack

Berkowitz Pleads Innocent; Plans Insanity Defense

Report Carter Picks Ala. Judge As FBI Chief

- The possibility of a second "compressional" wave in porous media was suggested in a paper published in 1944 by Yakov Frenkel.
- Biot, in a paper in 1956, mentions the existence of Frenkel's paper, but says little of substance about it. In the same paper, Biot gave a heuristic analysis that also led to the suggestion that there was a second compressional wave.
- If there were any experimental discoveries of such a second wave in porous media prior to 1956, they were not announced (at least not noticeably) in the main-stream scientific literature.

- 

A beginning along the present lines was made by Frenkel.[2] He discusses the rotational and dilatational waves, but the subject is summarily treated and important features are neglected.

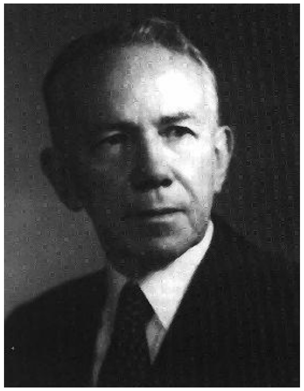
Sound absorption in material containing air was the object of extensive work by Zwikker and Kosten.[3] Rotational waves are not considered, and simplified equations are used for the dilatational waves. The

[2] J. Frenkel, J. Phys. (U. S. S. R) 8, 230 (1944).
[3] C. Zwikker and C. W. Kosten, *Sound Absorbing Materials* (Elsevier Publishing Company, Inc., New York, 1949).

- This may be a case where theory preceded experiment.

Biot's Field Equations

(low frequency version, 1956a)



solid equation

$$\frac{\partial^2}{\partial t^2}\left(\rho_{11}\boldsymbol{u}+\rho_{12}\boldsymbol{U}\right)-\boldsymbol{\nabla}\left(A'\boldsymbol{\nabla}\cdot\boldsymbol{u}+Q\boldsymbol{\nabla}\cdot\boldsymbol{U}\right)+\boldsymbol{\nabla}\times\{N\left(\boldsymbol{\nabla}\times\boldsymbol{u}\right)\}=b\frac{\partial}{\partial t}\left(\boldsymbol{U}-\boldsymbol{u}\right)$$

inertia term *pressure term* *shear term* *dashpot term*

$$A'=A+2N$$

fluid equation

$$\frac{\partial^2}{\partial t^2}\left(\rho_{21}\boldsymbol{u}+\rho_{22}\boldsymbol{U}\right)-\boldsymbol{\nabla}\left(Q\boldsymbol{\nabla}\cdot\boldsymbol{u}+R\boldsymbol{\nabla}\cdot\boldsymbol{U}\right)=-b\frac{\partial}{\partial t}\left(\boldsymbol{U}-\boldsymbol{u}\right)$$

$$\rho_{12}=\rho_{21}$$

2 vector fields

8 heuristic coefficients

Identifications derived from modified Burridge-Keller formulation:*

**Derivation presented at ASA meeting in June 2006.*

- $\boldsymbol{u}_{\mathrm{Biot}} = \langle \boldsymbol{u} \rangle_s$
- $\boldsymbol{U}_{\mathrm{Biot}} = \langle \boldsymbol{U} \rangle_f$

- $\rho_{11} = \chi_s \rho_s + \phi \chi_f \rho_f$
- $\rho_{12} = -\phi \chi_f \rho_f$
- $\rho_{22} = (1+\phi)\chi_f \rho_f$
- $b = \dfrac{\eta \chi_f}{a^2 \beta}$
- $A = B_{\mathrm{eff}} \chi_s^2 \left[1 - \gamma_{\mathrm{dil}}\right]^2 + \gamma_{\mathrm{dil}} \chi_s \left(\lambda_L + \frac{2}{3}\mu_L \right) - \frac{2}{3}\mu_L \chi_s \gamma_{\mathrm{sh}}$
- $N = \mu_L \chi_s \gamma_{\mathrm{sh}}$
- $R = B_{\mathrm{eff}} \chi_f^2$
- $Q = B_{\mathrm{eff}} \chi_s \chi_f \left[1 - \gamma_{\mathrm{dil}}\right]$

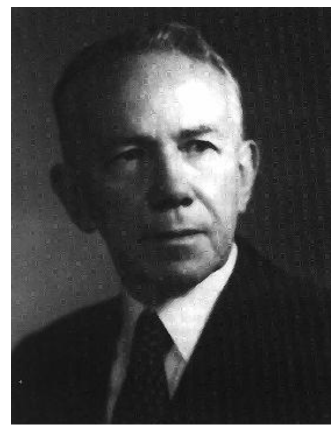
χ_f	*porosity*
$\chi_s + \chi_f = 1$	
$\langle\ \rangle$	*averaged over microscale*
$\beta,\ \phi,\ \gamma_{\mathrm{dil}},\ \gamma_{\mathrm{sh}}$	*dimensionless parameters*
η	*viscosity*
a	*representative grain size*

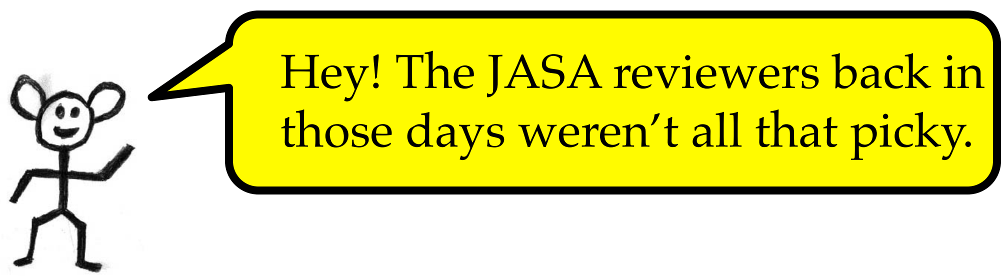
Effective bulk modulus

$$B_{\mathrm{eff}} = \left[\frac{3\chi_s (1 - \gamma_{\mathrm{dil}})}{3\lambda_L + 2\mu_L} + \frac{\chi_f}{\rho_f c_f^2} \right]^{-1}$$

(cf. Mallock, ProcRoySoc, 1910)

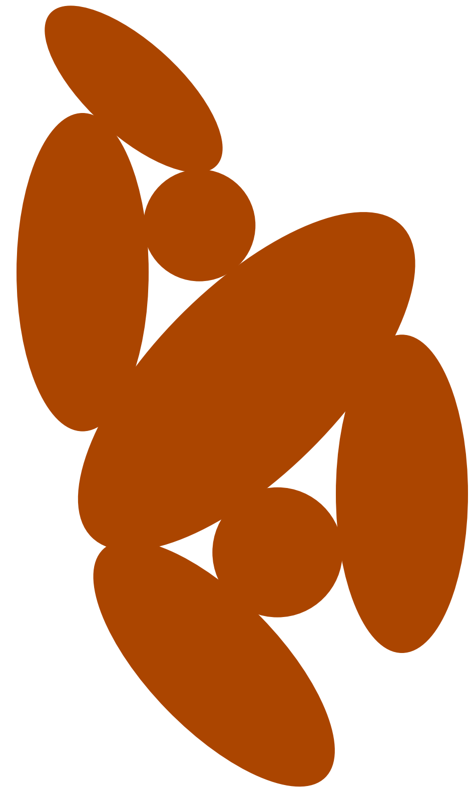
Biot and Frenkel never explicitly mentioned the concept of ***local averaging*** in their papers, but the idea appears to be implicit.



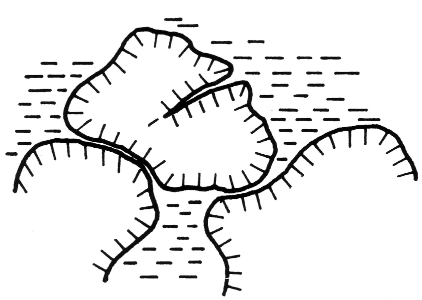


General observations about

$$\beta,\ \phi,\ \gamma_{\rm dil},\ \gamma_{\rm sh}$$



- *Dimensionless*
- *Independent of porosity*
- *Independent of material properties*
- *Depend on grain shapes, grain orientation, size distribution*
- *Very difficult to compute numerically*



(Figure from Biot, JASA, 1962)

Dispersion relation for Biot's two ``longitudinal" waves

$$\begin{bmatrix} -\omega^2\rho_{11} + k^2(2N+A) - i\omega b & -\omega^2\rho_{12} + k^2 Q + i\omega b \\ -\omega^2\rho_{12} + k^2 Q + i\omega b & -\omega^2\rho_{22} + k^2 R - i\omega b \end{bmatrix} \begin{Bmatrix} \hat{u} \\ \hat{U} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\boxed{\alpha k^4 - \left(\beta\omega^2 + i\epsilon b\omega\right) k^2 + \left(\gamma\omega^4 + i\delta b\omega^3\right) = 0}$$

quadratic equation with frequency-dependent coefficients

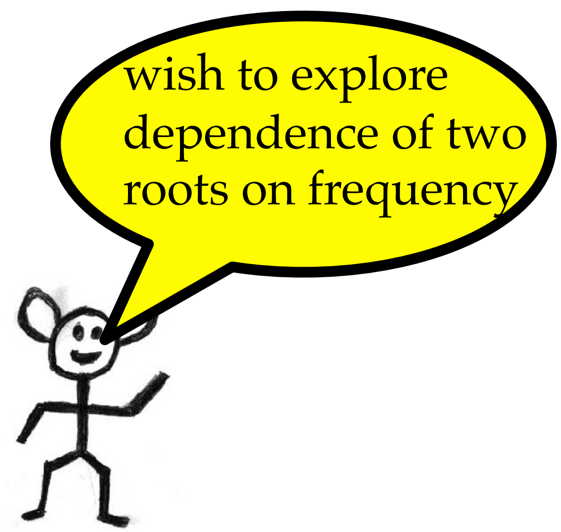
$$\alpha = (2N+A)R - \underline{Q^2}$$

$$\beta = R\rho_{11} + (2N+A)\rho_{22} - \underline{2Q\rho_{12}}$$

$$\gamma = \rho_{11}\rho_{22} - \underline{\rho_{12}^2}$$

$$\delta = \rho_{11} + \underline{2\rho_{12}} + \rho_{22}$$

$$\epsilon = 2N + A + \underline{2Q} + R$$



Frequency dependences of wave-numbers

presuming

$\omega_3 > 0$

(three characteristic frequencies defined in subsequent slide)

$$(k_1)^2 = \frac{i\omega\epsilon b}{2\alpha}\left[1 - i\frac{\omega}{\omega_1} - \left(1 - \frac{\omega^2}{\omega_2^2} - 2i\frac{\omega}{\omega_3}\right)^{1/2}\right]$$

$$(k_1)^2 \to \frac{\epsilon b}{2\alpha}\left[\left(\frac{1}{\omega_1} - \frac{1}{\omega_3}\right)\omega^2 + i\frac{1}{2}\left(\frac{1}{\omega_2^2} + \frac{1}{\omega_3^2}\right)\omega^3\right]$$

(low frequency limit)

$$(k_2)^2 = \frac{i\omega\epsilon b}{2\alpha}\left[1 - i\frac{\omega}{\omega_1} + \left(1 - \frac{\omega^2}{\omega_2^2} - 2i\frac{\omega}{\omega_3}\right)^{1/2}\right]$$

$$(k_2)^2 \to \frac{\epsilon b}{\alpha}\left[i\omega + \left(\frac{1}{\omega_1} + \frac{1}{\omega_3}\right)\omega^2\right]$$

(low frequency limit)

$$(k_1)^2 = \frac{i\omega\epsilon b}{2\alpha}\left[1 - i\frac{\omega}{\omega_1} - \left(1 - \frac{\omega^2}{\omega_2^2} - 2i\frac{\omega}{\omega_3}\right)^{1/2}\right]$$

$$\omega_1 = \frac{\epsilon b}{\beta} = b\left[\frac{(2N+A)R - Q^2}{R\rho_{11} + (2N+A)\rho_{22} - 2Q\rho_{12}}\right]$$

$$\omega_2^2 = \frac{b^2\epsilon^2}{\beta^2 - 4\alpha\gamma} = b^2\left[\frac{(2N+A+R+2Q)^2}{(R\rho_{11} - [2N+A]\rho_{22})^2 + 4Q^2\rho_{11}\rho_{22} + 4\rho_{12}^2(2N+A)R}\right]$$

$$\omega_3 = \frac{2b\epsilon}{\beta\epsilon - 4\alpha\delta}$$

$$\omega_2 > \omega_1$$

$$\alpha = (2N+A)R - Q^2$$

$$\delta = \rho_{11} + 2\rho_{12} + \rho_{22}$$

$$\beta = R\rho_{11} + (2N+A)\rho_{22} - 2Q\rho_{12}$$

$$\epsilon = 2N + A + 2Q + R$$

Fast and slow waves

presuming

$\omega_3 > 0$

$$(k_2)^2 - (k_1)^2 = \frac{i\omega\epsilon b}{\alpha}\,[\text{Radical}]$$

$$[\text{Radical}] = \left(1 - \frac{\omega^2}{\omega_2^2} - 2i\frac{\omega}{\omega_3}\right)^{1/2}$$

$$-\pi < \text{Ph}\{[\text{Radical}]^2\} < 0$$

$$-\pi/2 < \text{Ph}\{[\text{Radical}]\} < 0$$

$$[\text{Radical}] = a - ib$$

$$\text{Re}\left\{\left(k_2^2\right)\right\} > \text{Re}\left\{\left(k_1^2\right)\right\}$$

Conclude: if both waves are predominantly propagating waves, the second wave has the smaller phase velocity.

Orthogonality relation

$$\begin{bmatrix} -\omega^2\rho_{11} + k^2\left(2N + A\right) - i\omega b & -\omega^2\rho_{12} + k^2 Q + i\omega b \\ -\omega^2\rho_{12} + k^2 Q + i\omega b & -\omega^2\rho_{22} + k^2 R - i\omega b \end{bmatrix} \left\{ \begin{array}{c} \hat{u} \\ \hat{U} \end{array} \right\} = \left\{ \begin{array}{c} 0 \\ 0 \end{array} \right\}$$

requires

$$\left\{ \begin{array}{c} \hat{u}_2 \\ \hat{U}_2 \end{array} \right\}^T \begin{bmatrix} 2N + A & Q \\ Q & R \end{bmatrix} \left\{ \begin{array}{c} \hat{u}_1 \\ \hat{U}_1 \end{array} \right\} = 0$$

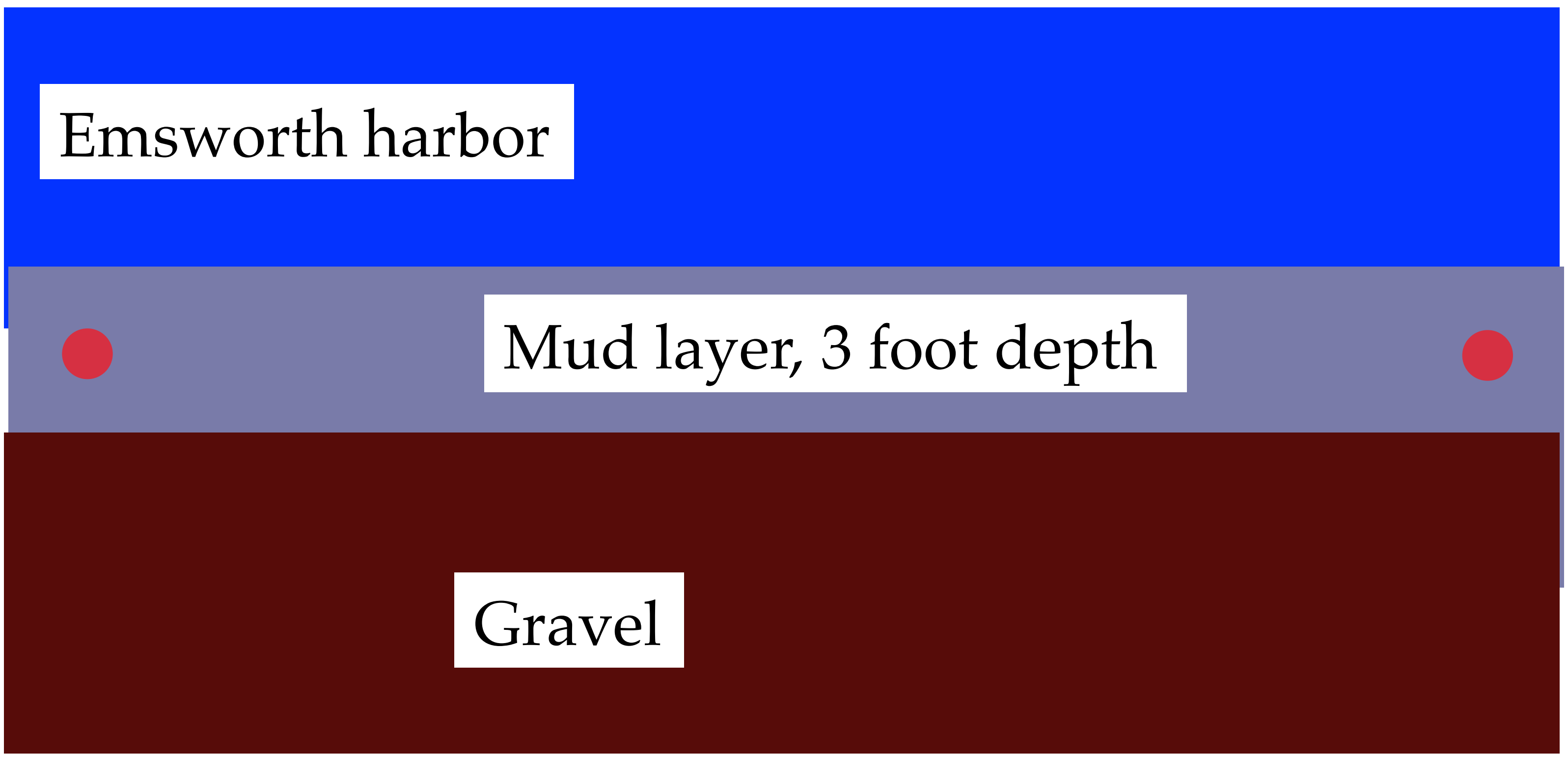
Conclude: if $\dfrac{\hat{U}_1}{\hat{u}_1}$ *is real and positive,*

then $\dfrac{\hat{U}_2}{\hat{u}_2}$ *is real and negative.*

Caveat: what you might call the fast wave is not necessarily all that fast!

A. B. Wood and D. E. Weston, The Propagation of Sound in Mud, Acustica, Vol. 14, 1964.

The velocity of sound measured in situ was 3% below that of sea water.



Acoustic-mode wave speed

$$c_{\mathrm{ac}}^2 = \frac{A + 2N + 2Q + R}{\rho_{11} + 2\rho_{12} + \rho_{22}}$$

$$\boxed{c_{\mathrm{ac}}^2 = \frac{B_{\mathrm{eff}}\left(1 - \chi_s \gamma_{\mathrm{dil}}\right)^2 + \frac{4}{3}\mu_L \chi_s \gamma_{\mathrm{sh}} + \gamma_{\mathrm{dil}} \chi_s \left(\lambda_L + \frac{2}{3}\mu_L\right)}{\chi_s \rho_s + \chi_f \rho_f}}$$

(almost Wood's equation)

$$c_{\mathrm{ac}}^2 \approx \frac{B_{\mathrm{eff}}}{\rho_{\mathrm{eff}}} \qquad B_{\mathrm{eff}} = \left[\frac{3\chi_s\left(1 - \gamma_{\mathrm{dil}}\right)}{3\lambda_L + 2\mu_L} + \frac{\chi_f}{\rho_f c_f^2}\right]^{-1}$$

$$\frac{1}{B_{\mathrm{eff}}} \approx \chi_s \frac{1}{B_{\mathrm{s}}} + \chi_f \frac{1}{B_{\mathrm{f}}}$$

A. B. Wood and D. E. Weston, The Propagation of Sound in Mud, Acustica, Vol. 14, 1964.

The velocity of sound measured in situ was 3% below that of sea water.

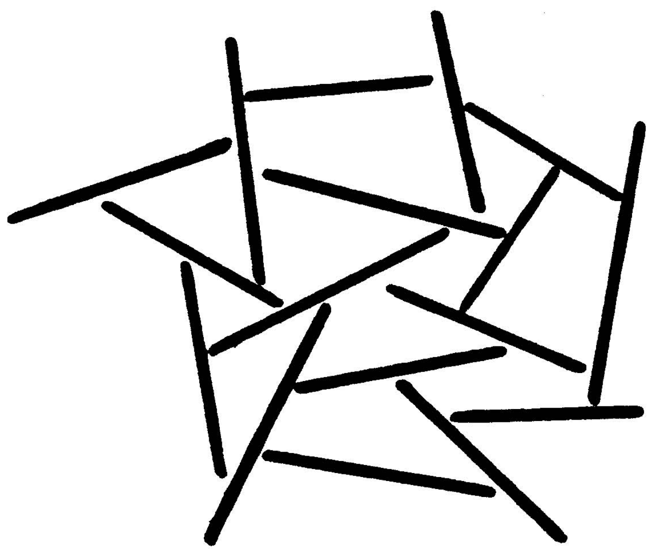
$$c_{\mathrm{ac}} \approx c_f \left\{ 1 - \frac{\chi_s}{2} \left(\frac{\rho_s}{\rho_f} + \frac{B_f}{B_s} - 2 \right) \right\} \qquad \text{for small } \chi_s$$

(positive)

$$0.03 \approx \frac{\chi_s}{2} \left(\frac{\rho_s}{\rho_f} + \frac{B_f}{B_s} - 2 \right)$$

$$\frac{\rho_s}{\rho_f} \approx 2.6 \qquad \frac{B_f}{B_s} \approx 0.1 \rightarrow \boxed{\chi_s \approx 0.1}$$

Why is the porosity of mud so high?



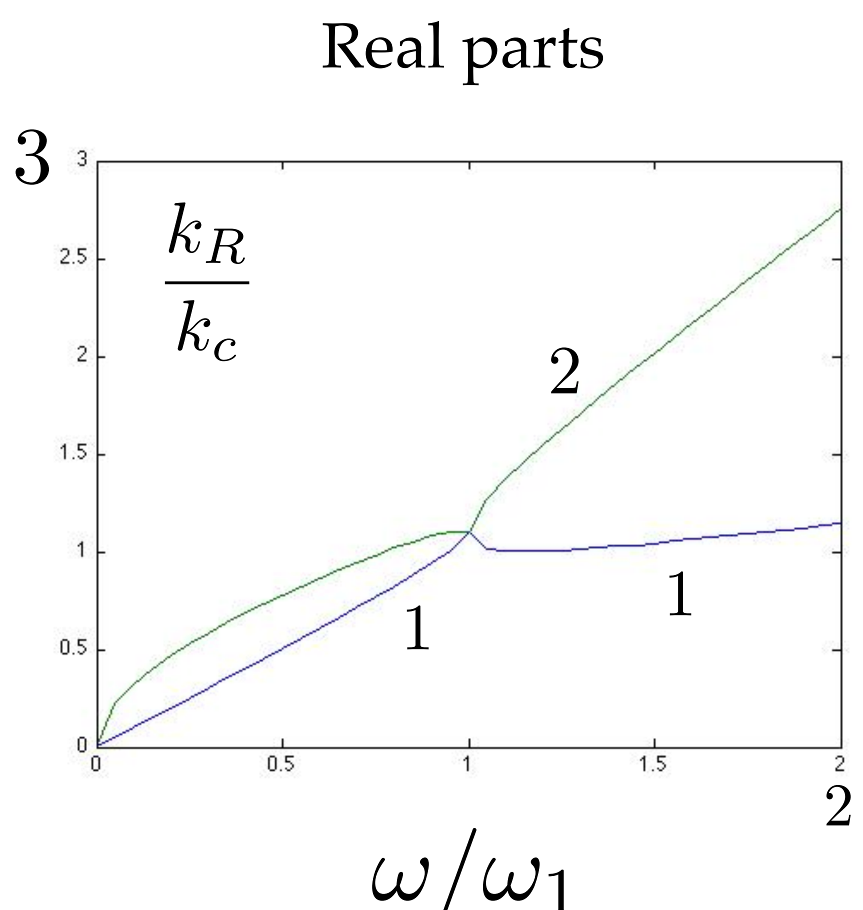
Card-house theory

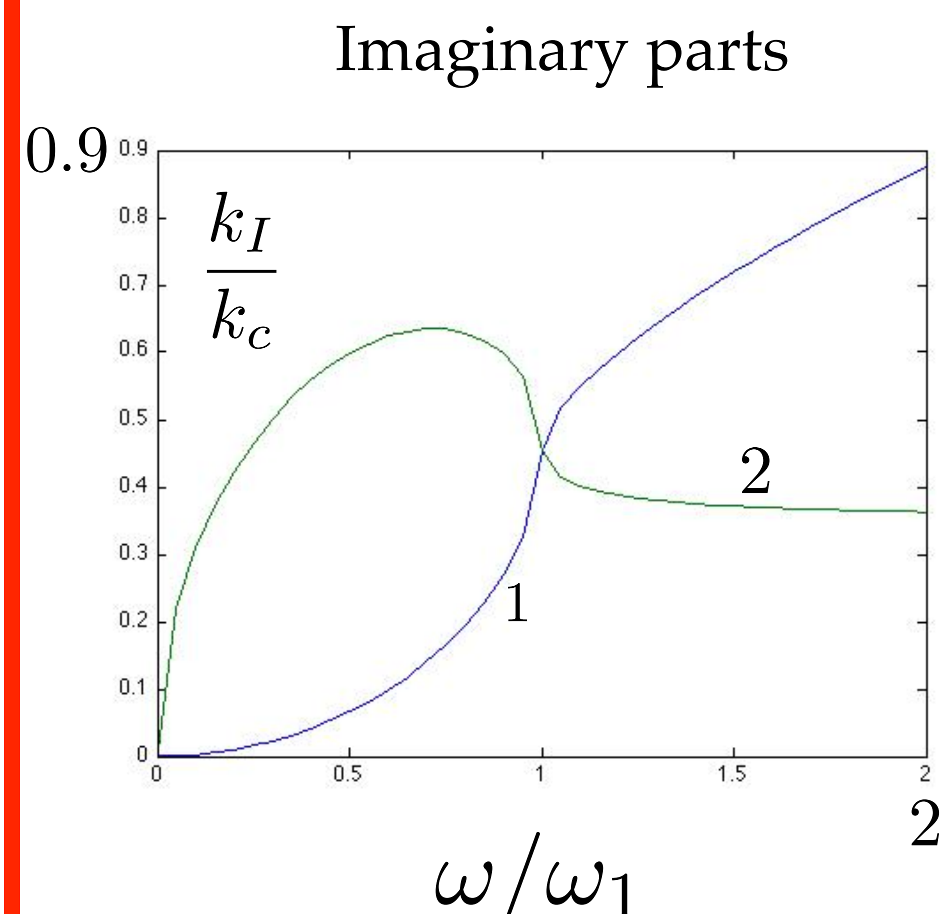
$$(k_1)^2 = \frac{i\omega\epsilon b}{2\alpha}\left[1 - i\frac{\omega}{\underline{\omega_1}} - \left(1 - \frac{\omega^2}{\underline{\omega_2^2}} - 2i\frac{\omega}{\underline{\omega_3}}\right)^{1/2}\right]$$

$$\boxed{\begin{array}{c} \omega_3 \gg \omega_2 > \omega_1 \\ \omega_2 \approx \omega_1 \end{array}}$$

Granular media limit:

$$\boxed{\begin{aligned} (k_1)^2 &\approx \frac{i\omega\epsilon b}{2\alpha}\left[1 - i\frac{\omega}{\omega_1} - \left(1 - \frac{\omega^2}{\omega_1^2} - i0^+\right)^{1/2}\right] \\ (k_2)^2 &\approx \frac{i\omega\epsilon b}{2\alpha}\left[1 - i\frac{\omega}{\omega_1} + \left(1 - \frac{\omega^2}{\omega_1^2} - i0^+\right)^{1/2}\right] \end{aligned}}$$





$$\frac{k_1}{k_c} \to e^{i\pi/4}\left[\left(\frac{\omega}{\omega_1}\right)^{1/2} - \frac{1}{4}\left(\frac{\omega_1}{\omega}\right)^{1/2}\right]$$

$$\frac{k_2}{k_c} \to \sqrt{2}\frac{\omega}{\omega_1} + i\frac{1}{2\sqrt{2}}$$

According to Biot’s formulation:

(In the granular media limit)

- *Both modes are “the same” at* $\omega \approx \omega_2$
- *Both modes are highly attenuated at this frequency*
- *First mode morphs into a diffusion wave*
- *Far above this frequency, the second mode is propagating.*
- *The first mode is not propagating at moderately high frequencies*

Questions about "mid-frequency" validity of Biot's formulation

(In the granular media limit)

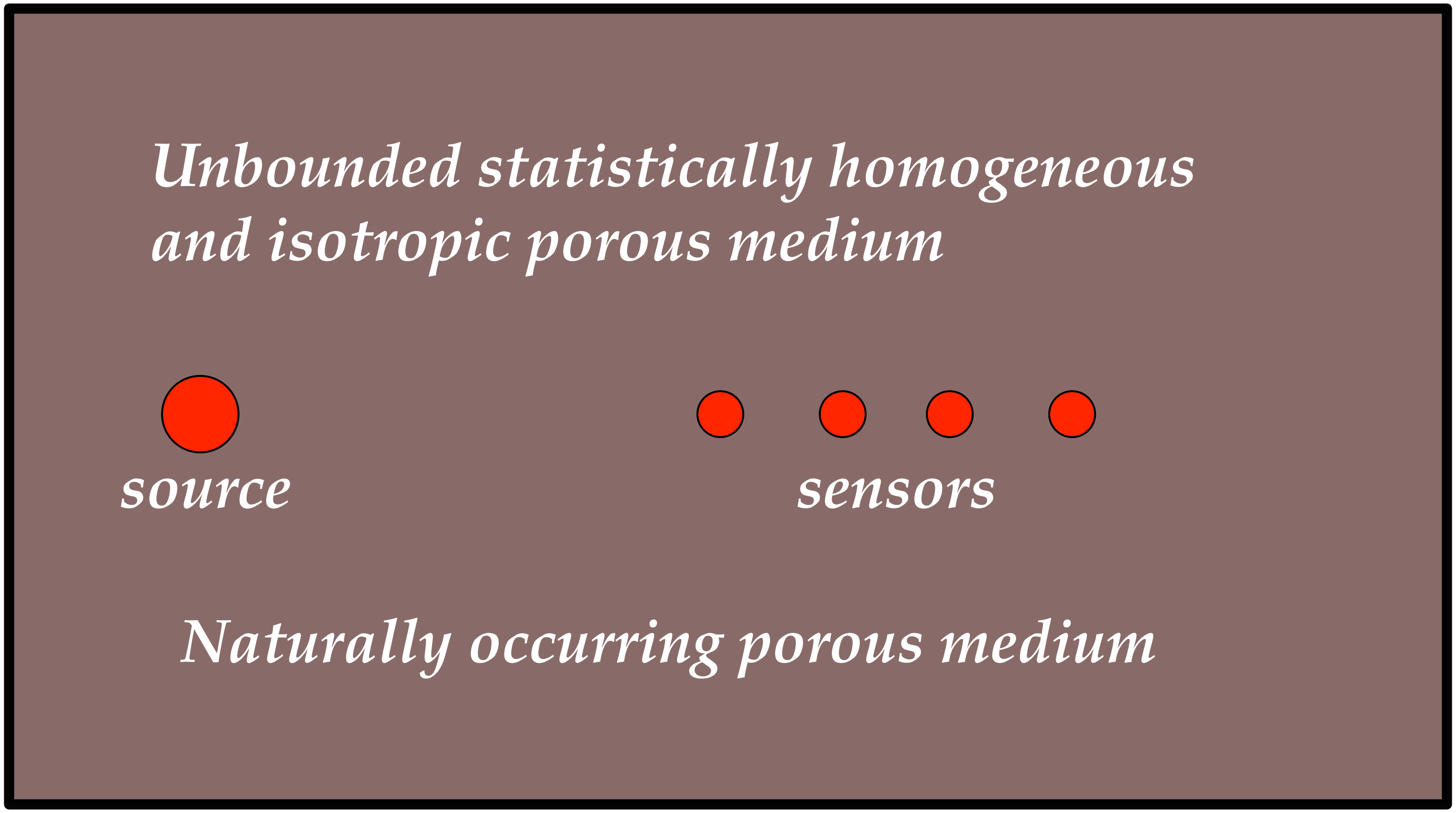
- ***Behavior near*** $\omega \approx \omega_2$ ***seems bizarre***
- ***No simple physical explanation***
- ***Is the higher frequency realization of the second mode the more appropriate extension of the lower frequency realization of the first mode?***
- ***If one of the two modes is not existing at higher frequencies, then perhaps it is the first mode that is more likely to be the one that isn't propagating.***

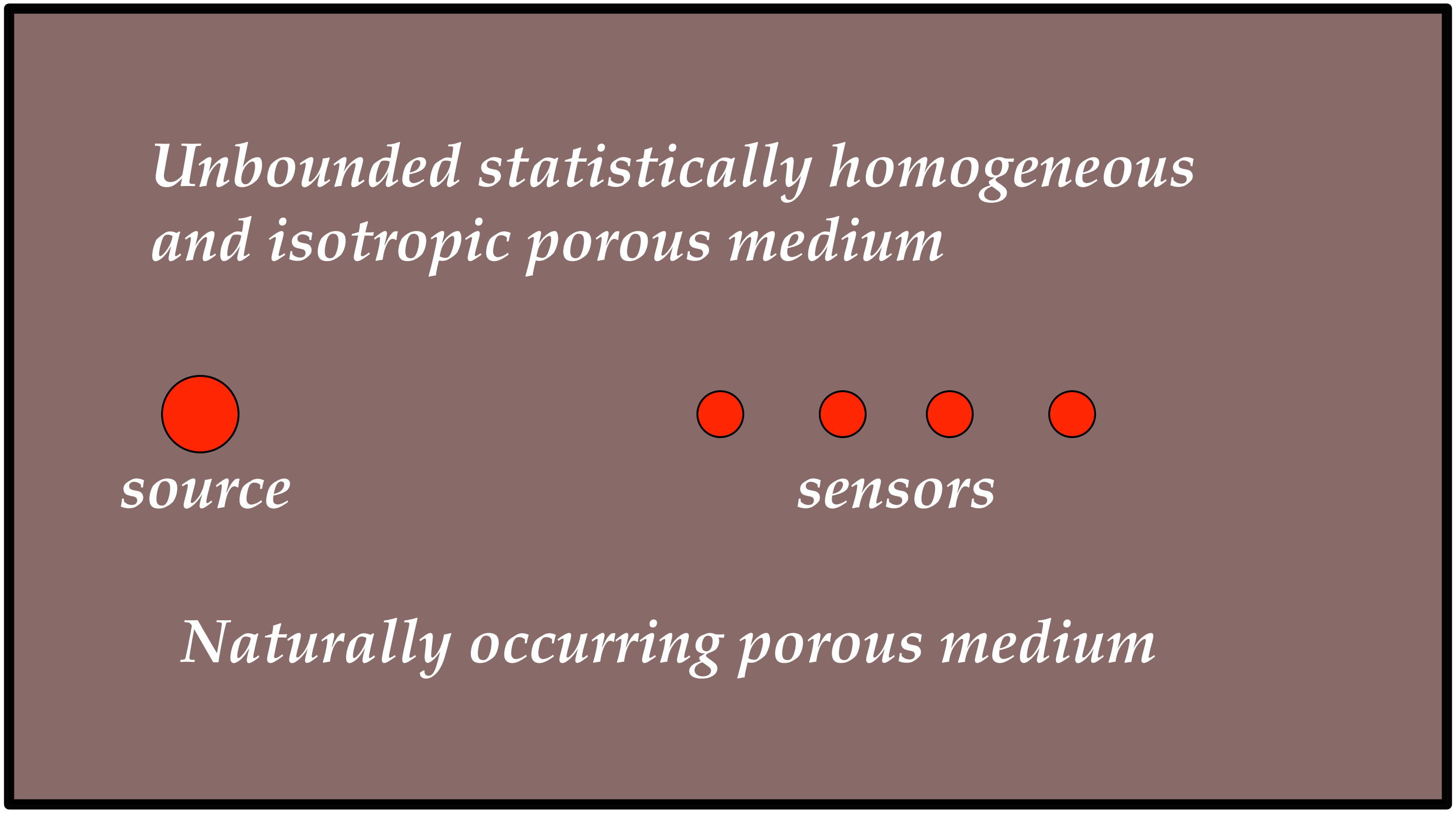
Remark in a paper published two years ago

[2] It is now over 50 years since *Biot* [1956a, 1956b] predicted a second, or slow, compressional wave that exists in addition to the normal “fast” compressional and the shear waves in saturated porous and permeable media. It is nearly 30 years since *Plona* [1980] experimentally verified its existence. Despite this and also despite a large theoretical literature, there are still only a few definitive observations of the slow wave. The mode is highly attenuated; and it is not clear if it has yet been seen outside of the laboratory. However, our inability to find the slow wave directly does

Bouzidi and Schmitt, JGR, August 2009

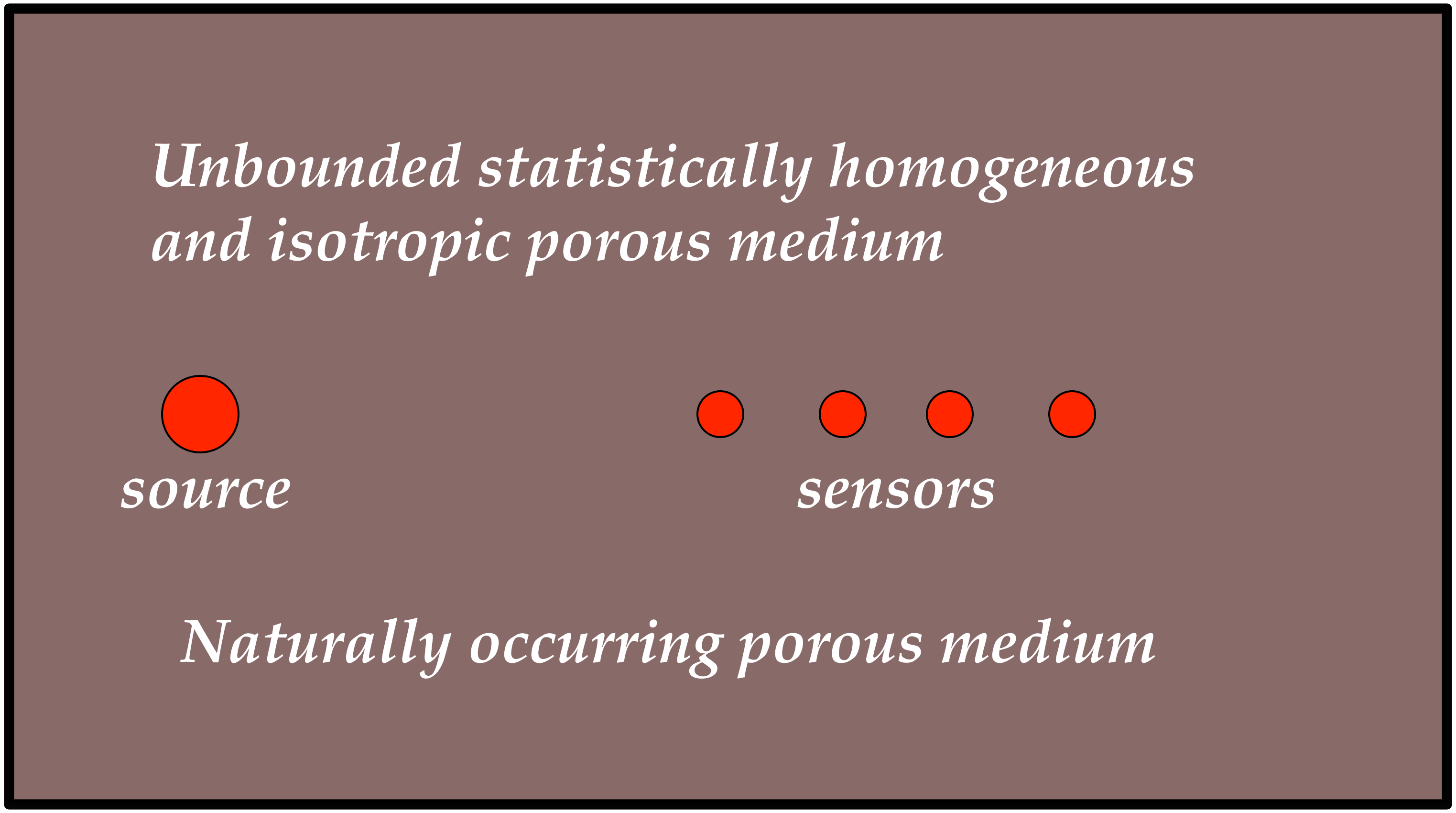
Ideal experiment: propagation in porous media





Source:

- Spatially concentrated
- Excites no shear
- Tone burst
- Narrow band of frequencies
- Finite time duration
- Several cycles

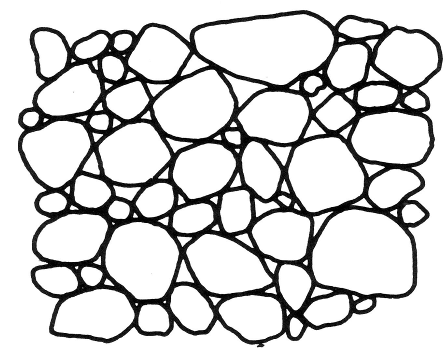


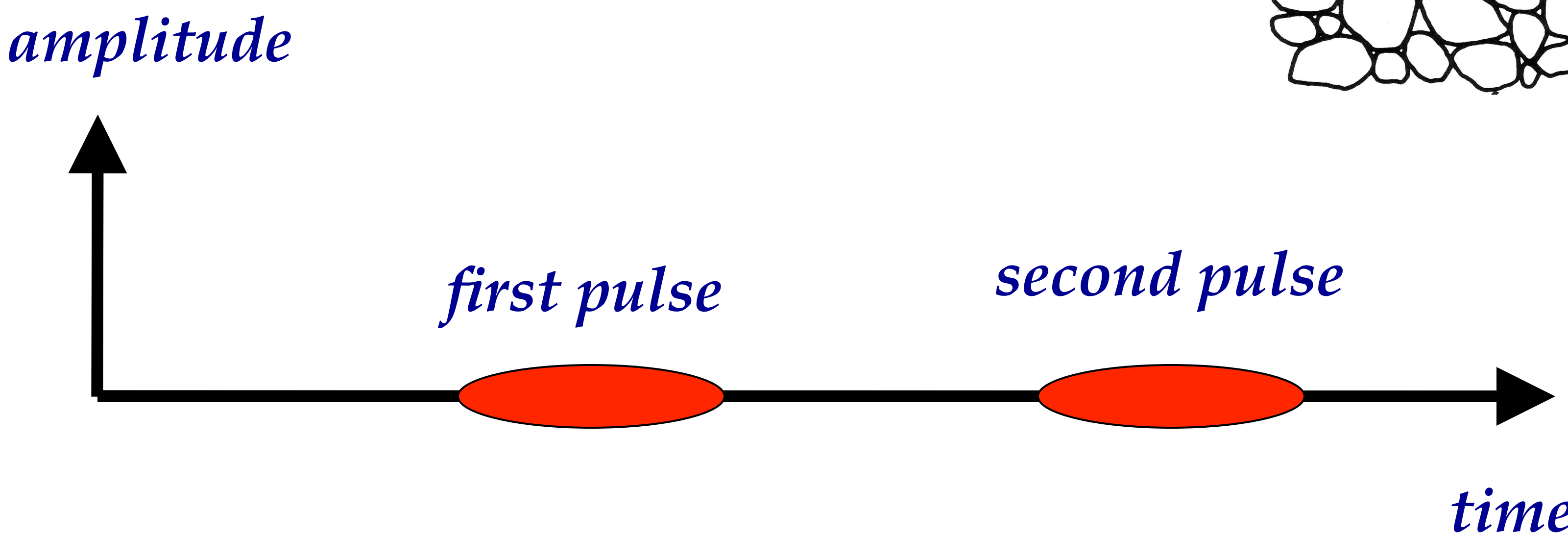
Sensors:

- Various large radial distances
- Measure time variation of quantities
- Locally spatially averaged
- Solid matter displacement
- Fluid matter displacement

"Commonly" expected outcome of ideal experiment

At fixed radial distance r:

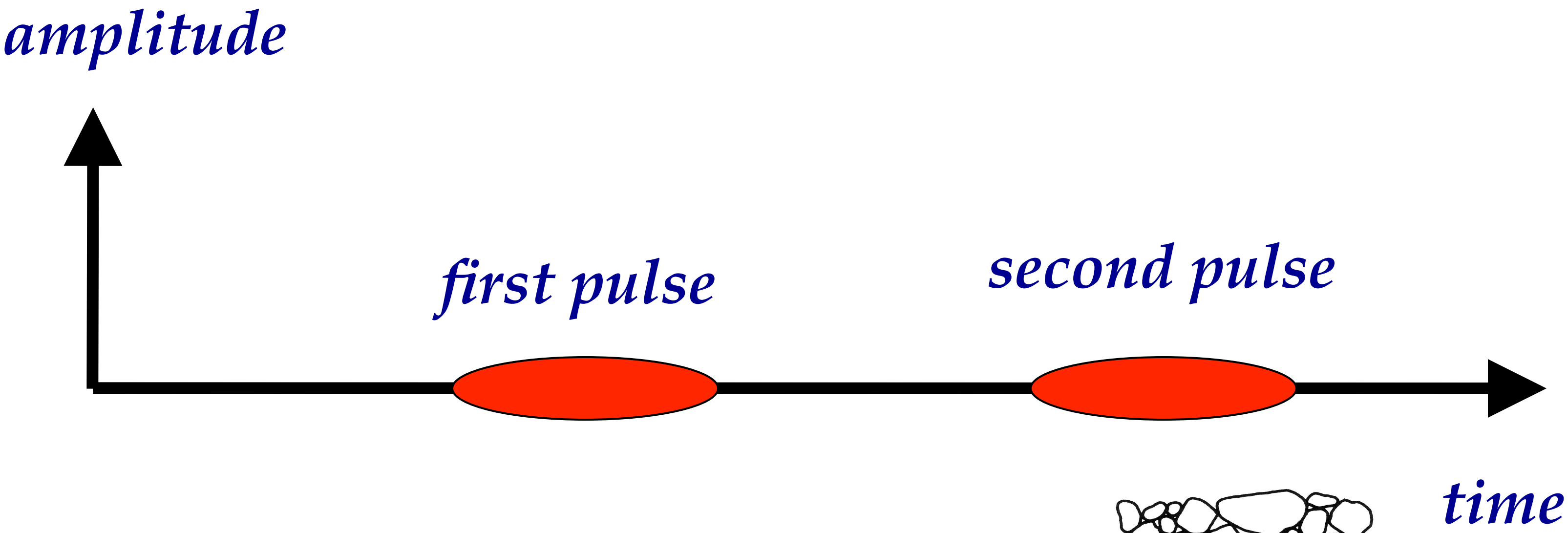


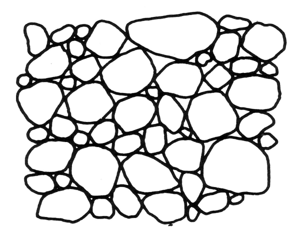


Data at different radial distances is expected to indicate that each pulse travels with a definite characteristic speed.

v_1 v_2

“Commonly” expected properties of the two pulses





- $\lambda = v/f$ is much larger than a representative grain size
- For first pulse, solid and fluid displacements in phase
- For second pulse, solid and fluid displacements out of phase
- “Energy” in pulse decreases with range nearly as 1 over range squared

$$\int_{\Delta t} (\mathrm{amp})^2 \, dt$$

Examination of derivation of Biot’s equations

The equations represent attempt(s) to provide a deterministic (as opposed to statistical) continuum mechanics (as opposed to kinetic theory) of porous media.

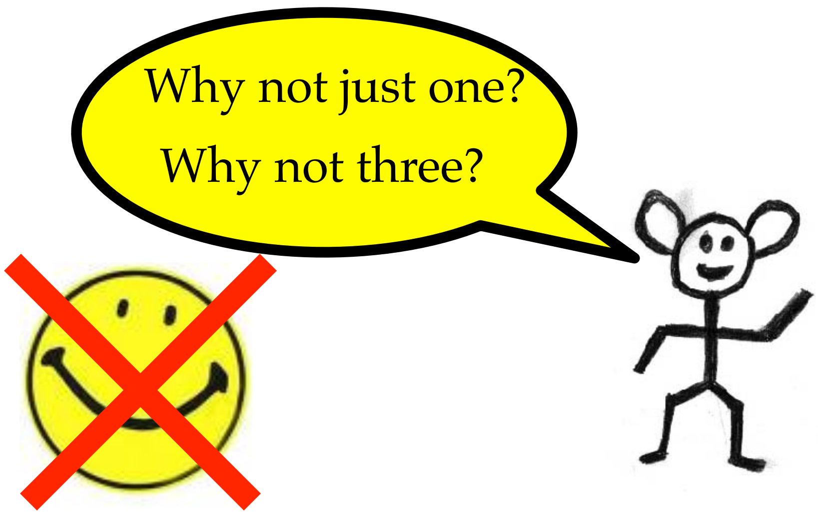
Biot wrote many similar papers and *changed notation frequently* in successive papers, and also *changed the form of his porous media equations* in successive papers.

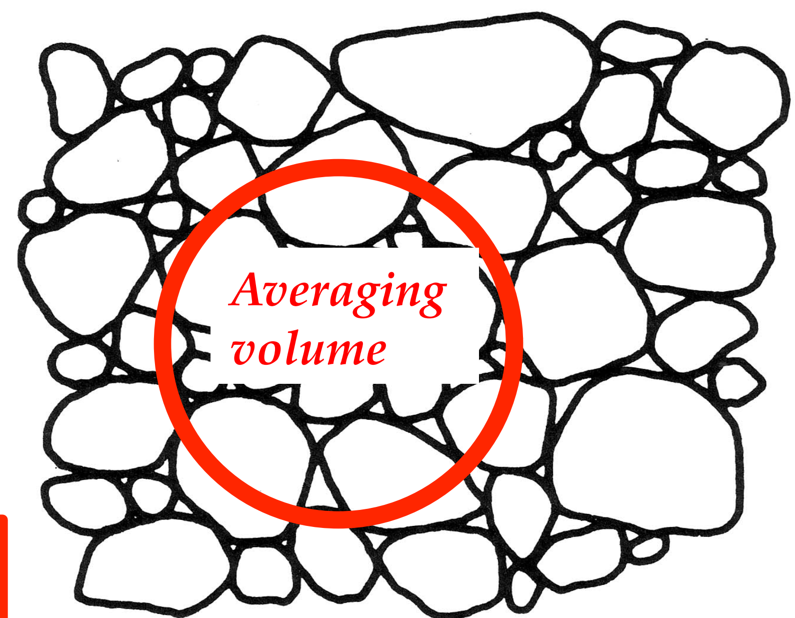
However, the changes in the underlying equations are *primarily cosmetic*. The underlying ***assumptions*** are, for the most part, the same in all the Biot papers.

What is the most crucial assumption and how universally applicable is it?

This is the crux of the present talk Obviously, the universal applicability is being questioned.

Assumption that ***two*** locally averaged displacement fields are ***adequate*** for description of any phenomenon of interest



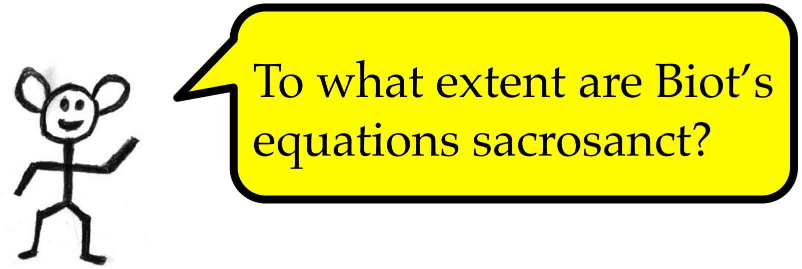


$\boldsymbol{u}$ *=local average of displacement vector of solid matter,*
averaged over the volume actually occupied by the solid

$\boldsymbol{U}$ *for fluid matter*

- Assumption dates back at least as far as Frenkel (1944)

$$\boxed{1 < 2 < \infty}$$



You have to have ***at least one*** displacement variable. Two is ***better*** than one, maybe three is ***better*** than two, maybe four is ***better*** than three. ***Stop*** at N if the resulting theory seems adequate.

Example for N > 2

$$\boldsymbol{u}_a, \boldsymbol{u}_b, \boldsymbol{u}_c$$

$$\boldsymbol{U}_A, \boldsymbol{U}_A, \boldsymbol{U}_C$$

$\boldsymbol{u}_a$ ***local average of displacement vector of solid matter that is in grains with characteristics of type `a', averaged over the volume occupied by solid matter of type 'a'.***

$\boldsymbol{U}_A$ ***local average of displacement vector of fluid matter that is in pores with characteristics of type `A', averaged over the volume occupied by fluid matter of type `A'.***

For Biot-Frenkel models, N is 2, and there are two `compressional waves'.

Second corresponds to diffusion disturbance in the limit of low frequencies.

Darcy diffusion mode

Henry Darcy

$$\boldsymbol{\nabla} \times \boldsymbol{U}_{\mathrm{D}} = 0; \qquad \boldsymbol{\nabla} \times \boldsymbol{u}_{\mathrm{D}} = 0$$

$$\boxed{\nabla^2 \left(\boldsymbol{U}_D - \boldsymbol{u}_D\right) = \kappa \frac{\partial}{\partial t} \left(\boldsymbol{U}_D - \boldsymbol{u}_D\right)}$$

diffusion equation

$\boldsymbol{U}_D$ $\longrightarrow$ $\boldsymbol{u}_D$ $\longleftarrow$

oppositely directed

The prediction of a Darcy mode disturbance by the N=2 Biot model at low frequencies is *physically realistic*. And both it and the acoustic compressional mode cannot simultaneously be predicted by a N=1 model.

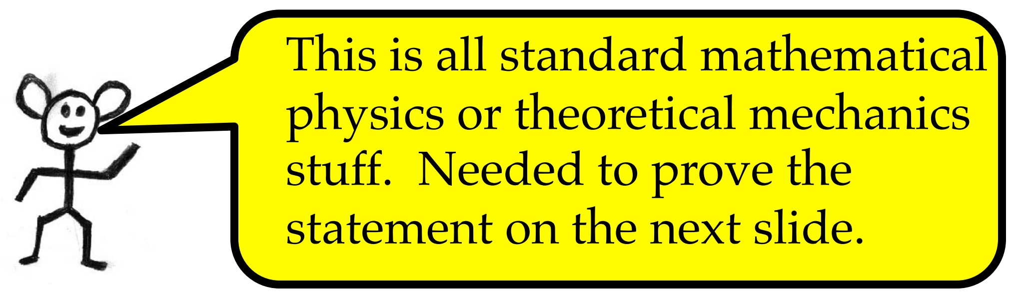
Very good!

OK. So having an N=2 model might give you something good beyond what you would get from an N=1 model.

But ---- what about the predictions at higher frequencies?

Analytical discovery of distinct wave types associated with a model with N different displacement vector fields

- N coupled vector (3-components) linear partial differential equations
- Derived from Hamilton's principle (extended Newtonian mechanics)
- Each equation of second order in spatial variables
- May involve operators that are complicated functions of d/dt
- Medium is statistically homogeneous and isotropic

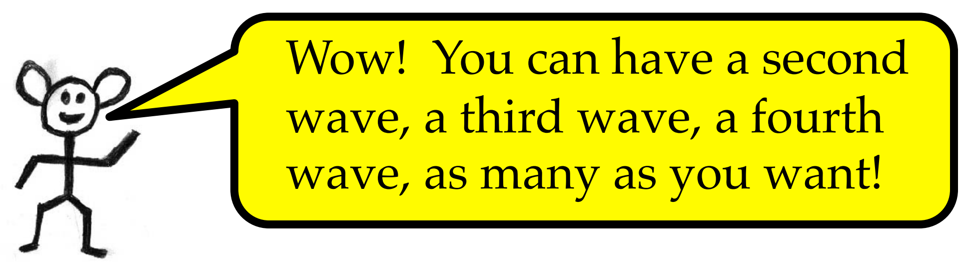


Polynomial of N-th degree has N roots

- For a continuum model with N vector displacement fields, you can have N longitudinal (compressional)`waves.'

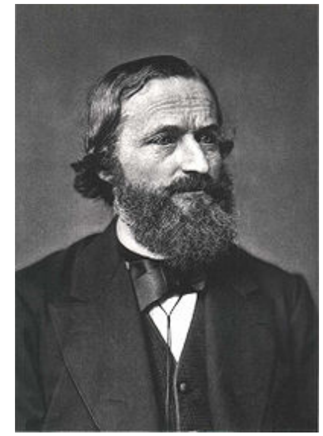
- N physically-admissible k_c 's

(some caveats)



Example of when N being larger results in an improvement

Rayleigh-Ritz technique for finding natural modes and frequencies



Gustav Kirchhoff

Expansion in terms of assumed modes

$$\Psi_{\mathrm{trial}} = \sum_{n=1}^{N} a_n \Psi_n(x, y)$$

$$[\omega^2]_{\mathrm{stationary}} = \text{Rayleigh quotient}$$
$$= \text{functional of } \Psi_{\mathrm{trial}}$$

John W. Strutt
(aka Lord Rayleigh)

Walter Ritz

The bigger N, the better the approximate answer for

$$\omega_1 \quad \text{and} \quad \Psi_{\mathrm{nat},1}$$

But ---- don't expect anything good for

$$\omega_N \quad \text{and} \quad \Psi_{\mathrm{nat},N}$$

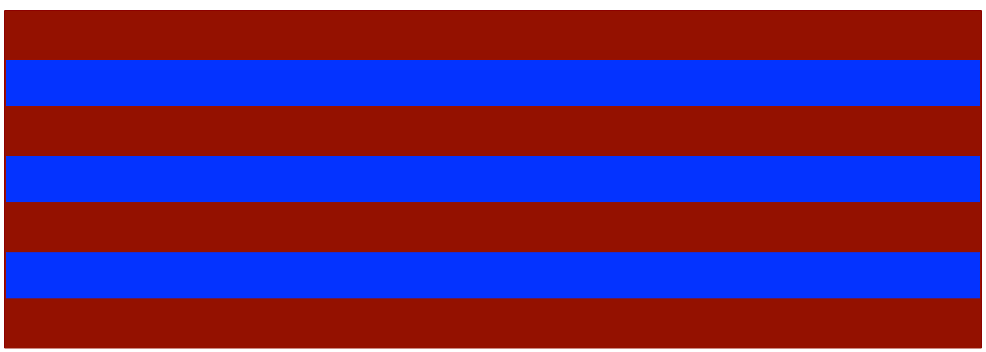
OK. Maybe some of the predictions for bigger N's should be taken with a grain of salt.

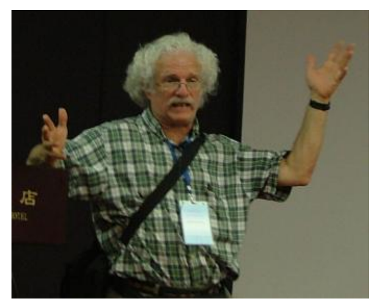
``Theoretical Proofs'' of Existence of Biot's Second Wave

[*Schoenberg APL 1983*] M. Schoenberg, Wave propagation in a finely laminated periodic elastoacoustic medium, *Appl. Phys. Letters*, Vol. 42, 1983.

[*Schoenberg JASA 1983*] M. Schoenberg and P. N. Sen, Properties of a periodically stratified acoustic half-space and its relation to a Biot fluid, *J. Acoust. Soc. Amer.*, Vol. 73, 1983.

However, the emergence of a second wave in these derivations requires that the medium be ***perfectly periodic.***





Michael Schoenberg
(1939-2008)

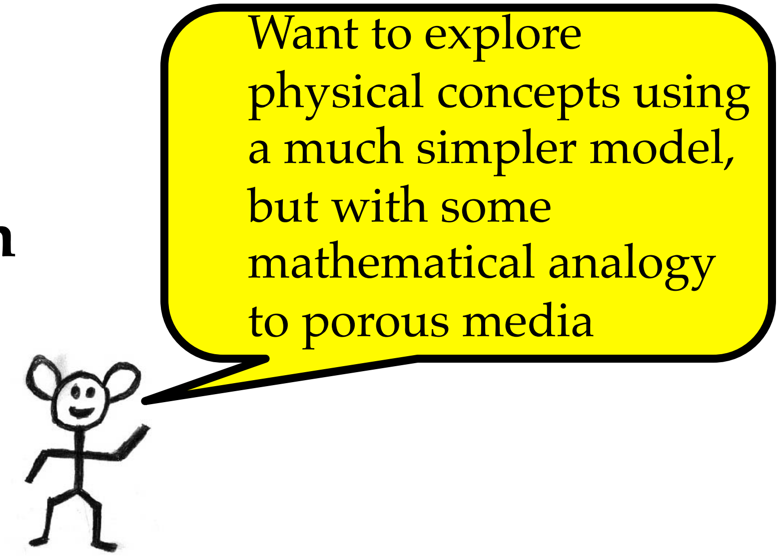
Is the medium being perfectly periodic essential?

- Naturally occurring porous medium is not perfectly periodic.
- It is typically a random medium.
- But it tends to be statistically homogeneous.
- Statistical descriptors for any one region tend to be same as for distant regions.

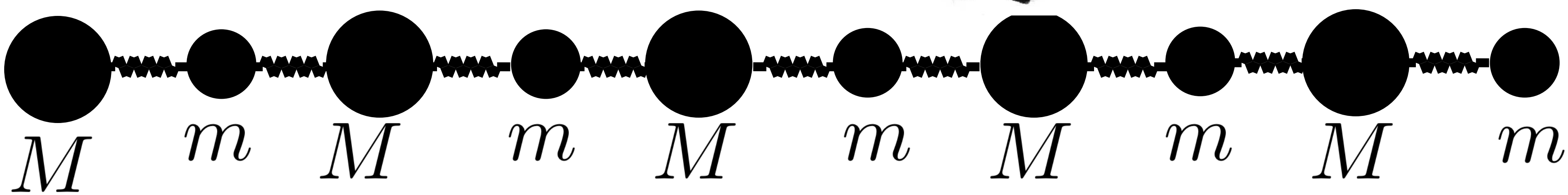
Max Born and Theodor von Karman

Über Schwingungen in Raumgittern

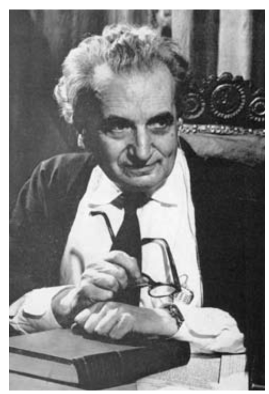
Physikalische Zeitung, 1912



(Linear chain of springs and masses.)



Max Born

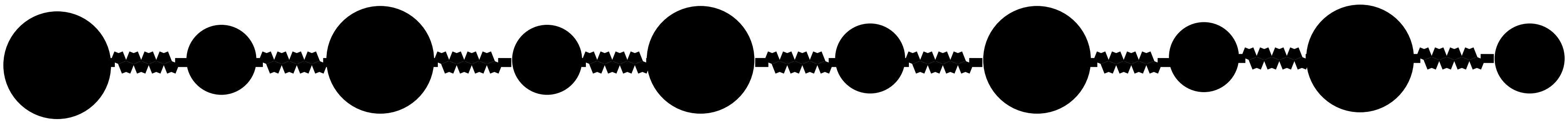


Theodor von Karman

Leon Brillouin

What happens when this lattice is not perfectly periodic?

Diatomic lattice example of Born and Von Karman (1912)



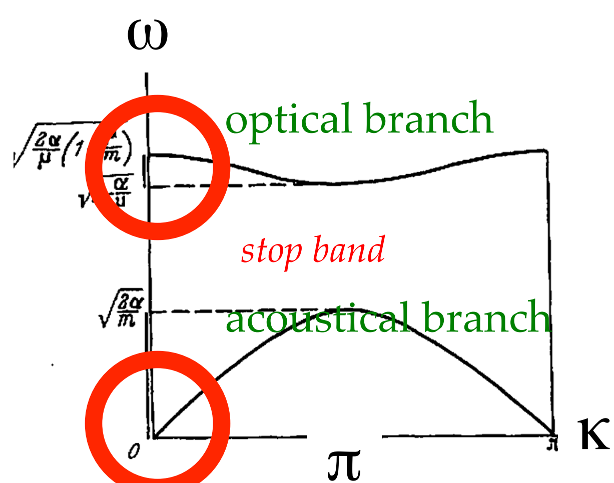
$$X_n = \hat{X} e^{-i\omega t} e^{i\kappa n} \qquad x_n = \hat{x} e^{-i\omega t} e^{i\kappa n}$$

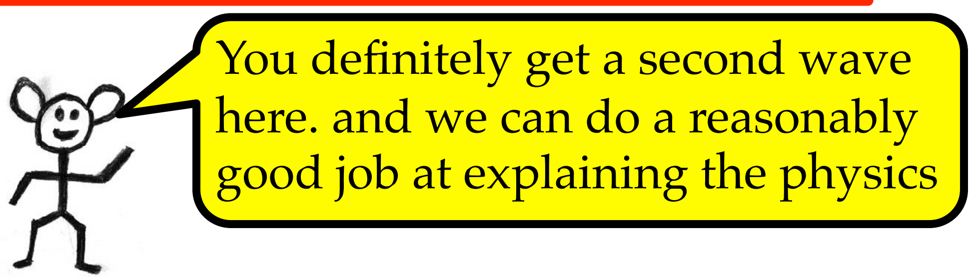
$$k = \frac{\kappa}{\Delta x}$$

$$\omega^4 - 2k_{\mathrm{sp}} \frac{M+m}{Mm}\omega^2 + \frac{4k_{\mathrm{sp}}^2}{Mm}\sin^2(\kappa/2) = 0$$

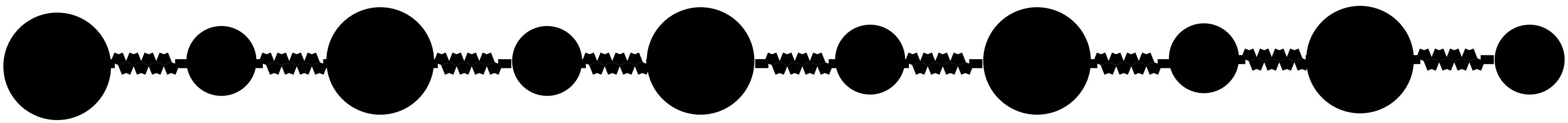
For small κ (long wavelengths):

$$\omega^4 - 2k_{\mathrm{sp}} \frac{M+m}{Mm}\omega^2 + \frac{k_{\mathrm{sp}}^2}{Mm}\kappa^2 = 0$$

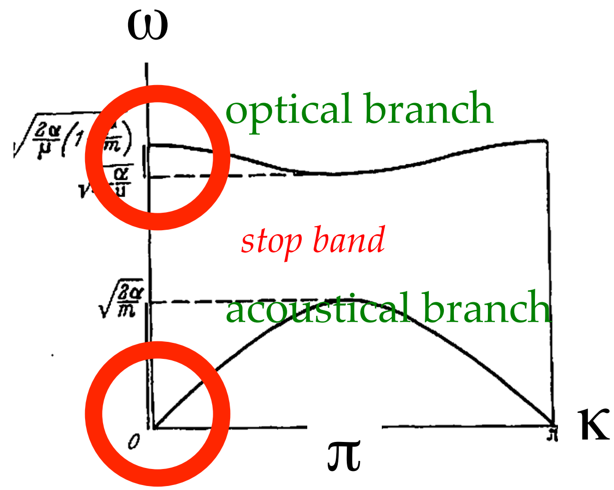




Physical interpretation of diatomic lattice model

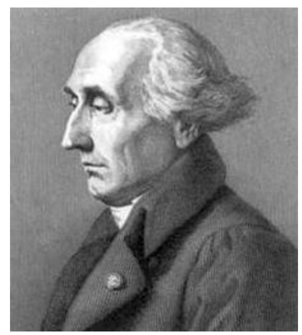


For small κ (long wavelengths):



acoustical branch

$$\kappa^2 \approx \frac{2(M+m)}{k_{\rm sp}}\omega^2$$



Joseph-Louis Lagrange

optical branch

$$\kappa^2 \approx \frac{2(M+m)}{k_{\rm sp}}\left[\frac{2k_{\rm sp}(M+m)}{Mm} - \omega^2\right]$$

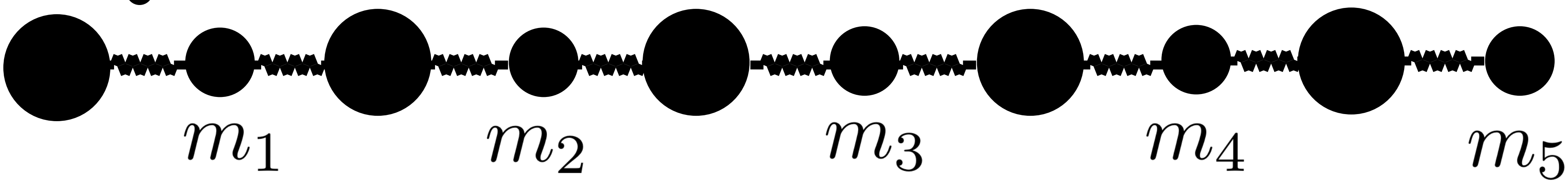
cutoff frequency squared

$$\boxed{\frac{\hat{X}}{\hat{x}} = -\frac{m}{M}}$$

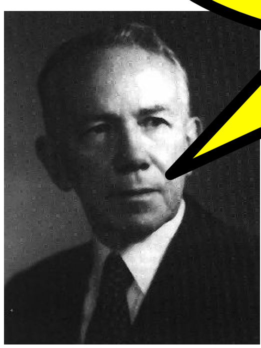
at cutoff frequency

Cutoff frequency is an internal resonance frequency!

Suppose all the little masses are not exactly the same



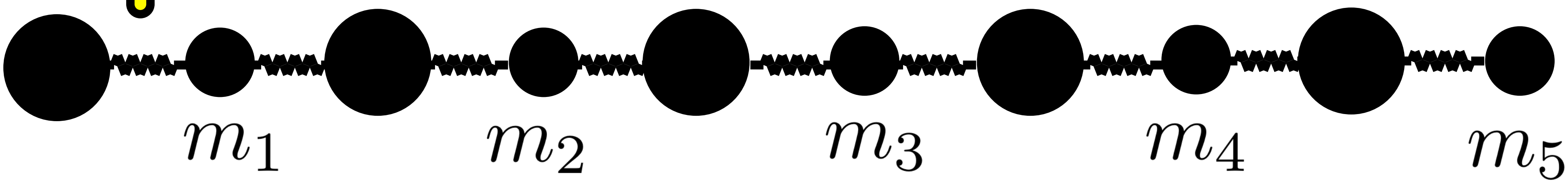
Find the average little mass
Replace each little mass
by the average mass



Take an arbitrarily long segment
of the chain and use periodic
boundary conditions

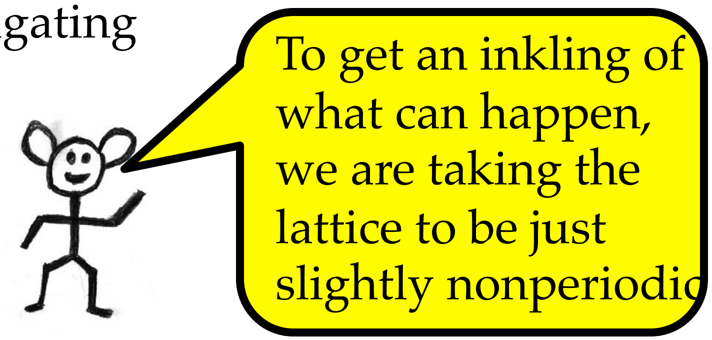
- N +1 little masses in a cell
- $m_{N+2} = m_1; \quad m_{N+3} = m_2$
- N optical modes
- Cutoff frequencies are nearly the same

Progressive interference of N optical modes



- Take a frequency for which all modes are propagating
- Suppose all modes are equally excited at x=0

$$\hat{p} = \frac{1}{N}\sum_{n=1}^{N} e^{ik_n x}$$



- Reduces to diatomic chain result when all little masses are the same
- With increasing propagation distance, different modes can interfere or reinforce. They all reinforce at x=0, so the initial trend is increasing interference.
- Plausible theory suggests that interference effect at large x causes average decrease as

$$|\hat{p}| \rightarrow K\frac{1}{\sqrt{x}}e^{ikx}$$

- Disturbance is no longer a plane wave.

Derivation of the `plausible' theoretical result

$$|\hat{p}| \to K \frac{1}{\sqrt{x}} e^{ikx}$$

$$\hat{p} = \frac{1}{N} \sum_{n=1}^{N} e^{ik_n x} \qquad k_1 < k_2 < \ldots < k_N$$

$$\hat{p} \approx \frac{1}{N} \int_0^N e^{ik_{\mathrm{sm}}(n)x} \, dn$$

Replacement of discrete wave-number series by smeared continuous function

$$\hat{p} \approx \left(e^{ik_{\mathrm{sm}}(n_{\mathrm{sp}})x} \right) \frac{1}{N} \int_C e^{i\alpha(\Delta n)^2} \, d(\Delta n)$$

Approximate evaluation of integral by method of stationary phase (sp)

$$\hat{p} \approx \left(e^{ik_{\mathrm{sm}}(n_{\mathrm{sp}})x} \right) \frac{e^{i\pi/4}}{N} \left(\frac{\pi}{\alpha} \right)^{1/2} \qquad \alpha = \frac{1}{2} k''_{\mathrm{sm}}(n_{\mathrm{sp}}) x$$

``Experimental Proofs'' of Existence of Biot's Second Wave

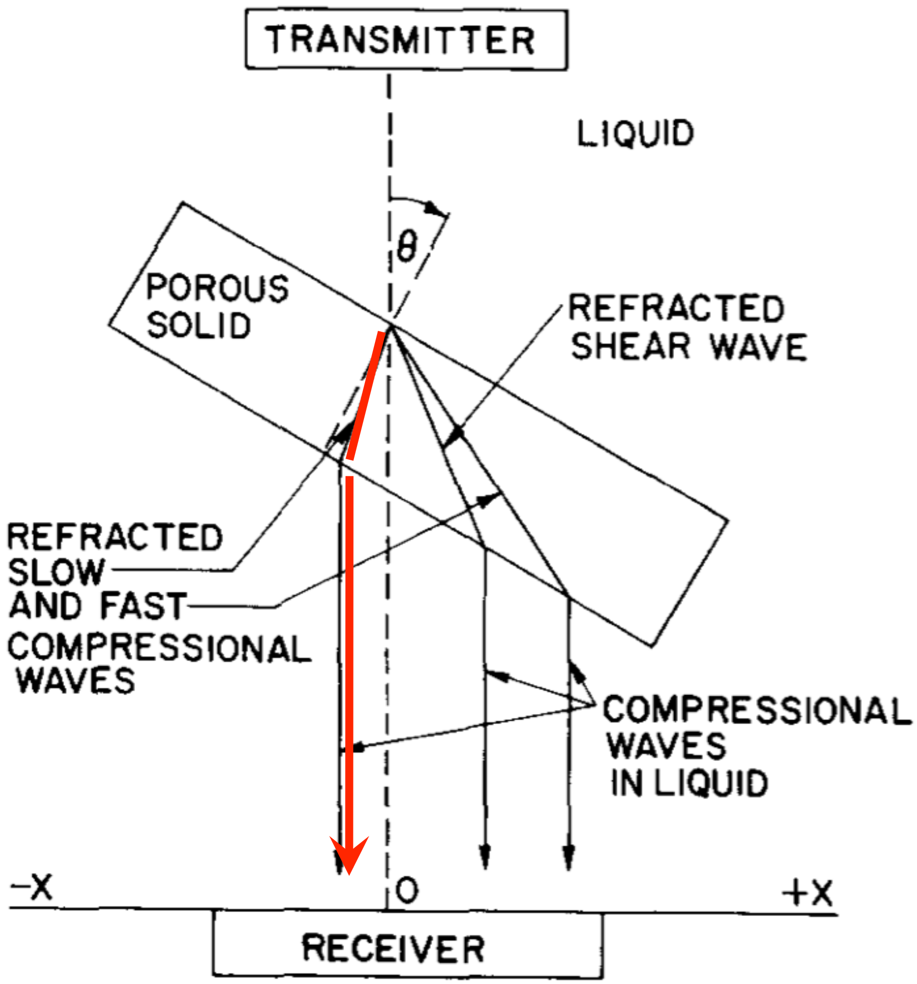
[***Plona APL 1980***] T. J. Plona, Observation of a second bulk compressional wave in a porous medium at ultrasonic frequencies, *Appl. Phys. Letters*, Vol. 36, 1980.

[***Johnson JASA 1982***] D. L. Johnson and T. J. Plona, Acoustic slow waves and the consolidation transition, *J. Acoust. Soc. Amer.*, Vol. 72, 1982.

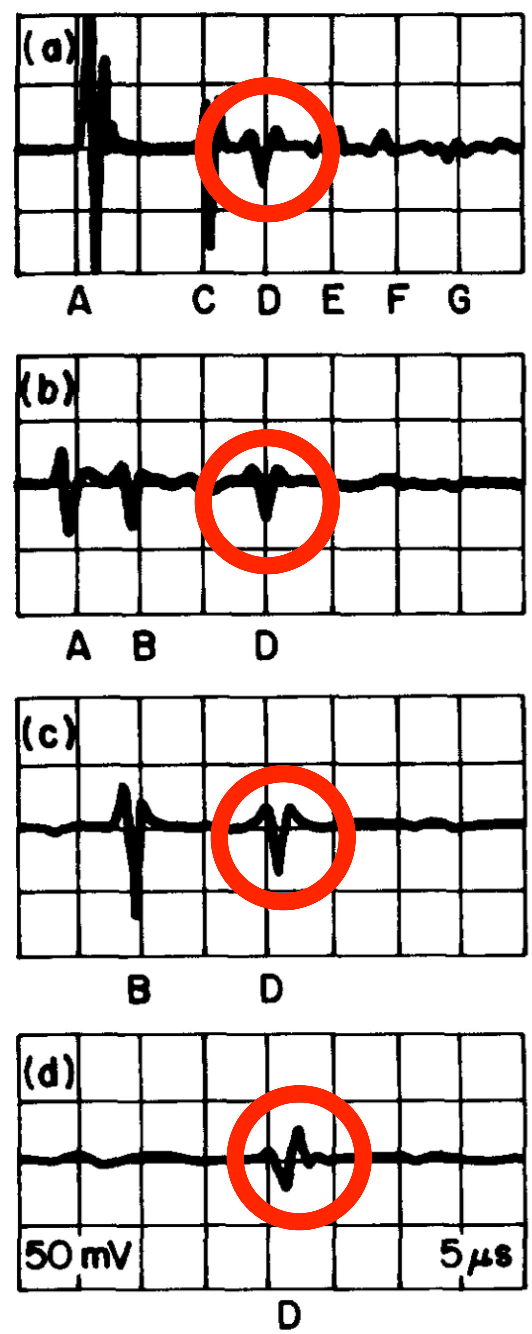
[***Plona JASA 1987***] T. J. Plona, K. M. Winkler, and M. Schoenberg, Acoustic waves in alternating fluid/solid layers, *J. Acoust. Soc. Amer.*, Vol. 81, 1987.

[***Johnson JAP 1994 II***] D. L. Johnson,T. J. Plona, and H. Kojima, Acoustic probing porous media with first and second sound. II. Acoustic properties of water-saturated porous media, waves in alternating fluid/solid layers, *J. Appl. Phys.*, Vol. 78, 1994.

Plona's experiment (1980)



mode-conversion technique

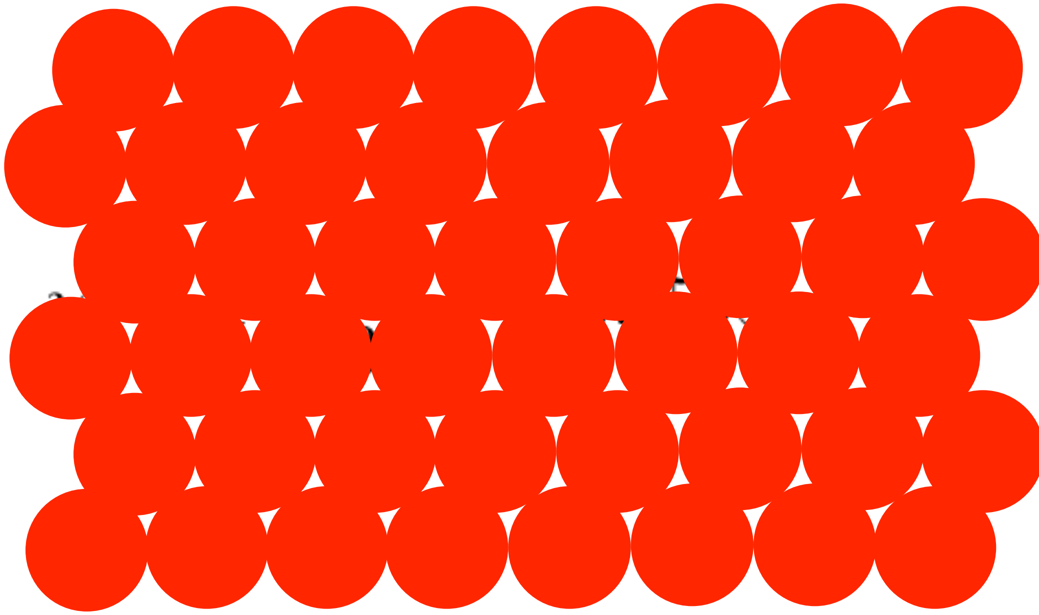


General comments about Plona's experiment

- Very difficult
- Clever technique

- Somewhat contrived, artificial mediun
- Medium nearly perfectly periodic

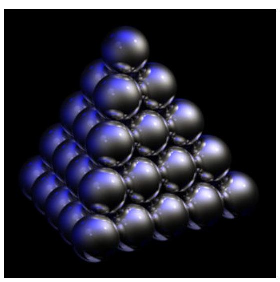
- Propagation distance very short
- Incomplete
  - *Only one frequency*
  - *Transition frequency range not explored*
  - *Plane wave nature not verified*



Plona constructed slab of sintered glass beads, all of nearly same diameter, reported porosity of the order of 0.258

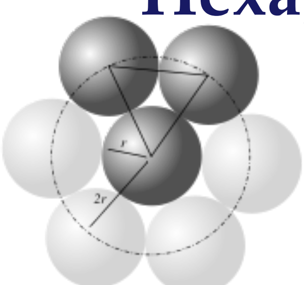
Hence, medium was probably nearly perfectly periodic.

Face centered cubic packing

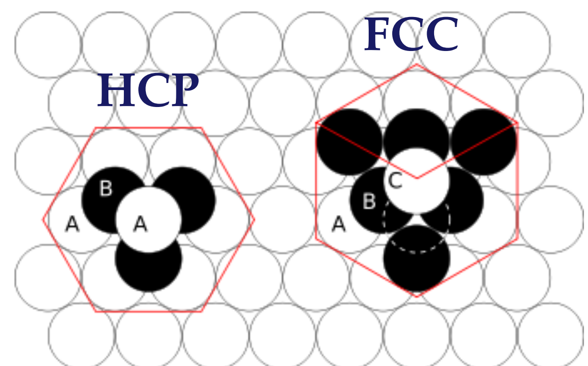


$$\chi_f = 1 - \frac{\pi}{3\sqrt{2}} = 0.2595$$

Hexagonal close packing

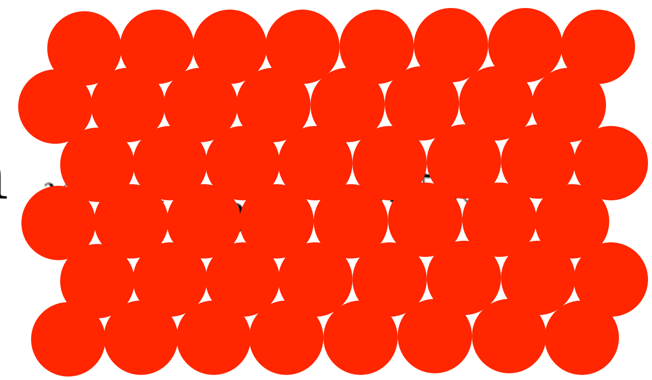


$$\chi_f = 1 - \frac{\pi}{3\sqrt{2}} = 0.2595$$



Parameters of Plona's Experiment

- Speed of second wave $c_2 = 1040\,\mathrm{m/s}$
- Diameter of glass beads $d = 0.21\ \text{to}\ 0.29\,\mathrm{mm}$
- Primary frequency of pulses $f = 500\,\mathrm{kHz}$
- Wavelength $\lambda = 2\,\mathrm{mm}$
- Thickness of slabs $h = 14\ \text{to}\ 21\,\mathrm{mm}$
- Relevant ratios
  - $\dfrac{h}{d} = 40\ \text{to}\ 200$
  - $\dfrac{\lambda}{d} = 6\ \text{to}\ 10$
  - $ka \approx 0.4$
  - $\dfrac{h}{\lambda} = 7\ \text{to}\ 10$



Too small for the detection of interference effects

Literature is now vast, but the present speaker is convinced that, at least for granular media in water, *you don't get the second pulse*.

What you should find instead is a ***lower amplitude tail*** that follows the first pulse, which has ***little spatial correlation*** and which is more representative of propagation through a random medium

Concluding Remarks

- Propagating second wave has no viable physical interpretation. ***(Very bad sign!)***
- Propagating second wave has not been unambiguously observed in naturally occurring porous media (random media). ***(Bigfoot sightings; Loch Ness monster sightings, Elvis sightings)***
- Experimental proofs of existence are incomplete, inconclusive, and not at all convincing

- Can devise new experiments that will more nearly adequately test this aspect of Biot's theory
- Numerical simulations at microscale to test theory are also conceivable
- Possible analogy to light waves diffusing through thick water clouds in atmosphere
- If dynamical disturbances other than sound or diffusion are observable and significant, then *development of statistical theory* is suggested