Propagation of acoustic waves in a one-dimensional macroscopically inhomogeneous poroelastic material.

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This abstract is derived from an article[1] recently accepted in J. Acoust. Soc. Am..

The study of wave propagation in macroscopically inhomogeneous porous media was initially motivated by: i) the design of sound absorbing porous materials with optimal material and geometrical property profiles and ii) the retrieval of the spatially varying material parameters of industrial foams. It is assumed that the wavelengths are larger than the average heterogeneity size at the pore scale so that the physical properties are homogenized. However, these properties can vary with the observation point within the material at the macroscopic scale of the specimen. Macroscopically inhomogeneous poroelastic materials imply that both acoustic and elastic properties are space-dependent at the macroscopic scale. As pointed out by several authors, the generalized formulation of the Biot theory[2] is suitable to the study of macroscopically inhomogeneous porous media. It can also take into account anisotropy and viscoelastic frames. The wave equation in macroscopically inhomogeneous porous media was derived from this alternative formulation[3] and solved in the case of rigid frame inhomogeneous porous materials via the Wave Splitting method and "transmission" Green's functions approach or via an iterative Born approximation procedure based on the specific Green's function of the configuration. The recovery of several profiles of spatially varying material parameters by means of an optimization approach, was then achieved still in the rigid frame approximation.

When saturated by a light fluid such as air, the frame is moving below the solid/fluid decoupling frequency. Moreover, in many applications porous materials are saturated by a heavy fluid such as water or bone marrow. They can also be excited mechanically. In these configuration, the rigid frame approximation is not valid and the full macroscopically inhomogeneous poroelastic model should be used and solved.

The constitutive linear stress-strain relations and the momentum conservation law in the absence of body forces are recalled for an inhomogeneous poroelastic material. These equations are then solved for a one-dimensional macroscopically inhomogeneous poroelastic material via the the state vector formalism together with Peano series. In the frequency domain, it consists in the rewriting of the constitutive linear stress-strain relations and the momentum conservation law in terms of the state vector, whose components can be chosen arbitrarily as long as they are continuous along the inhomogeneity of the material. These components are chosen to be adapted to the considered boundary problem. This leads after spatial Fourier transform to a system of first-order ordinary differential equations whose unknown is the state vector. The spatial dependence of the materials properties are accounted for through the spatially dependent matrix components. The solution of

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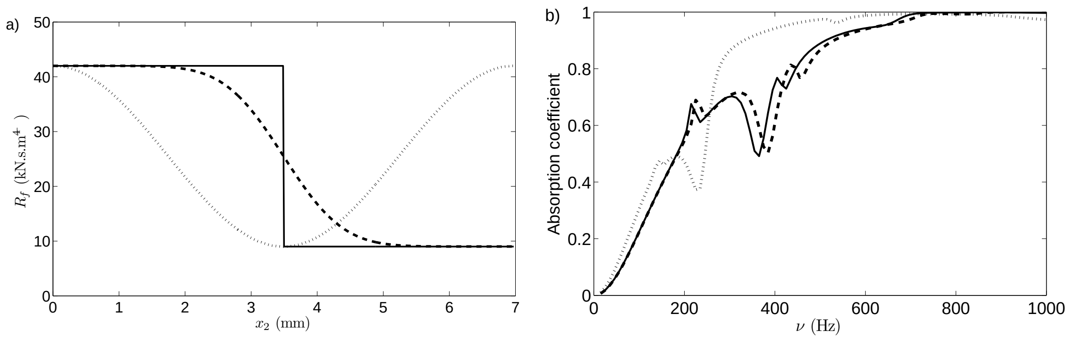

Figure 1: Example of the R_f profile considered a) and the corresponding absorption coefficient b): discontinuous profile (—), smoother profile (− − −), and Hanning profile (· · ·)

the system appears in terms of Peano series, which are well fitted for wave propagation problems in functionally graded materials. These series apply to continuously varying poroelastic properties and avoids problems related to a lack of discretization when the inhomogeneous materials are approximated by multilayers of homogeneous stratifications.

Numerical results obtained with this method are compared to calculations of the classical transfer matrix method performed with the MAINE3A code for a known two-layers porous material, considered as a single inhomogeneous layer. The transfer matrix method is particularly suitable to solve problems involving a layered configuration.

Numerical results are also compared to experimental measurements at normal incidence. The experiment consists in recording waves reflected by the chosen two-layered porous medium laid on the floor of a semi-anechoic room when excited at normal incidence by a dipolar source.

Finally, applications in material design for engineering applications are treated, by comparing the absorption coefficient of a macroscopically inhomogeneous porous plate with various inhomogeneity profiles. Figure1 depicts the profile generated for the flow resistivity R_f, as an example, for a discontinuous, a smooth and a Hanning profile, together with the corresponding absorption coefficient $A = 1 - |R|^2$. The shear modulus N, the bulk modulus of the closed porosity system λ_c, the elastic coupling coefficient α, the additional elastic parameter M, the porosity ϕ, the tortuosity τ_∞, the viscous and thermal characteristic lengths Λ and Λ', and the thermal resistivity R'_t also vary with an identical profile. The absorption coefficient is one of the most important parameters when designing a foam. These last years, this coefficient has received a large attention, particularly the attempt of increasing its value at low frequencies.

References

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