Convergence aspects in finite element solutions of multi-layered, heterogeneous porous materials

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To predict the propagation of elasto-acoustic waves through a porous medium, for an arbitrarily complex arrangement of different materials, geometries, boundary conditions and multilayer structures, it is necessary to resort to some kind of numerical solution procedure. For finite size problems, typical of interior sound proofing for passenger vehicles, such approximate solutions are mostly obtained through the use of the finite element method. Combining a mesh with a certain spatial resolution; often dictated by geometrical and topological variations, discontinuities in materials, boundaries, etc.; with a suitable choice of basis functions, the convergence of the numerical solution may at least in principle be ascertained in any given case.

However, there are a number of different aspects that need to be addressed to ensure the best possible efficacy in any given solution. Apart from the obvious and commonly known mesh and order of interpolation dependencies, there are other sources of inaccuracies that will be discussed in the current paper.

It is commonly known, that in the Biot model for isotropic, porous materials, three coupled waves are predicted by the theory, two compressional, dilatational waves, and one shear, distortional wave. Characteristics for these different waves, are the wave length, spatial decay rate, relative phase and amplitude between fluid and solid frame motion. In the paper, the convergence with respect to two of these will be discussed.

- The resolution of the respective wave lengths of the three waves, is a particularly interesting aspect especially in view of their rather large differences. The two compressional waves differ in their propagation speeds, one usually being slower than the other and thus having a shorter wave length. In addition the slow compressional wave is usually characterised by a fluid particle motion out of phase with the solid. This wave often exhibits quite rapid spatial decay of the wave amplitude with an associated near field type of behaviour and rather short wave lengths. Depending on the (an)elastic properties of a particular material, the shear wave is usually fairly lowly damped, at least for porous materials with low frame losses, but its wave speed is commonly quite low and the wave length comparably short.
- Common to all of these waves is the need for a finite element solution to resolve their main characteristics which are of importance for the response of the material. This is then particularly interesting close to discontinuities, where possibly short wave length, high dissipation with rapid spatial decay might occur. However, the lowly damped in-phase compressional wave is in many case responsible for most of the energy propagation through the material and, it is not obvious to which extent the highly damped waves must be resolved by the finite element mesh.

Another quite interesting aspect of the convergence and accuracy of a finite element solution is related to the variation of the material parameters between different layers in a multi-layered material combination. Large differences in the Young's modulus, may lead to loss of numerical precision due to cancellation effects in the linear equation solving process. For multi-layered, heterogeneous material combinations, where different materials may share common interfaces along intersections, care must be taken in the modelling in order to ensure an adequate convergence of the primary field variables.

These aspects will be illustrated in the lecture and their consequences and implictions in terms of the convergence of the solution using 3D finite element solutions of Biot’s equations based on higher order polynomial, hierarchical functions will be discussed.