Waveguide finite elements model and statistical energy analysis for multilayered structures comprising viscoelastic porous materials

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1 Abstract

This work presents a waveguide finite element model (WFEM) and a statistical energy analysis (SEA) for multilayered structures comprising viscoelastic porous materials described by Biot's theory [1, 2]. WFEM is an efficient and versatile numerical deterministic method suitable for structures showing constant properties in one or more *long* directions. The study of high-frequency vibrations in this kind of structures benefits from a description of the motion in terms of *waves*. WFEM, in fact, suits these requirements: its numerical efficiency allows to extend the analysis to higher frequencies than conventional finite elements; waves are its main output. This wave perspective is particular beneficial to the investigation of the sound reduction index of a multilayered structure. The characteristic wave types of such structure are shown by its dispersion relation, WFEM's main output, allowing a better understanding of its physics.

SEA is a high frequency method that intrinsically accounts for the statistical variation of industrial artifacts. It is very useful for virtual prototyping of highly developed products, and in engineering practise where often decisions are based on analyses that need not be too precise. SEA shows to perform very well with multilayered structures [3]: it catches their physics and reveals the main channels of vibroacoustic energy flow. The main difficulty in a SEA is the identification of the SEA elements, which may be elements of the vibroacustic response of the system under study. The dispersion relations calculated by the WFEM can be used for devising a SEA. First, they can be postprocessed to extract key SEA parameters such as modal density [4]. Second, they reveal the different wave types, and each of them may correspond to a SEA element [3].

The WFEM is derived from an extended Hamilton's principle, which is a natural variational principle also valid for dissipative media: the principle is based upon a self-adjoint functional that is stationary for the true motion of the system [5, 6]. Natural variational principles, actually, have appealing properties and advantages. They are "among the most beautiful of theoretical physics" [5] and are starting points for new formulations of mechanical structures [7]; they are excellent concepts for deriving equations of motions and natural boundary conditions [1, 8, 9]; and they are sound basis for new finite element formulations and techniques, as indicated by Finnveden *et al.*[8, 9, 10], and others [5, 6]; the Lagrangian's coordinate invariance allows the choice of any convenient set of generalized coordinates; Lagrange multipliers may be used to impose constraints on the functionals. This extended Hamilton's principle has been introduced by Morse and Feshbech. They propose to "consider, simultanously with the system having the usual friction, a *mirror image* system with negative frition into which the energy goes which is drained from the dissipative system. In this way the total energy is conserved and we have an invariant Lagrange function at the sacrifice of a certain amount of reality in some of the intermediate results."[11]. This principle has been successfully used in vibroacoustics by Gladwell [12], Morse and Ingard [13], and Finnveden, who extends the principle to cover also viscoelastic materials [9, 10] and to formulate WFEM models [8]. The extended Hamilton's principle is herein applied to porous materials, obtaining a Lagrangian in the displacement formulation and, via variational calculus, Biot's equations of motion.

Then, a *mixed* (u, p)-formulation employing the frame displacement and the pore fluid pressure as field variables is presented. Its Lagrangian is derived from the Lagrangian in the displacement formulation, utilizing its invariance and changing the generalised coordinates. This mixed formulation differs from Atalla *et al.*'s one [14]. In fact, both the presented displacement and mixed formulations are symmetric, conserve the Lagrangian and yield natural couplings to solid, fluid and porous media.

The WFEM herein presented benefits from these features. The dispersion relations calculated by the WFEM help devising a SEA for multilayered structures compraising Biot's porous materials. The performance of these numerical methods is verified with laboratories measurements of sound reduction index.

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