An improved multimodal method for sound propagation in ducts lined with bulk reacting linings

L. Kadiri*, W. Bi, Y. Aurégan

Laboratoire d'Acoustique de l'Université du Maine, UMR CNRS 6613,

Faculté des sciences, 72085 Le Mans, France

Over the years, the sound attenuation provided by ducts with bulk reacting lining has been widely investigated in the literature. Numerous models exist. Each of them presents assets and limitations according to the geometry of the studied problem. For a complete review of the methods, we refer to the recent articles of Kirby [7] and Panigrahi *et al.* [12] and the references therein.

The methods employed to study sound propagation in lined ducts can be categorized into two major types. On the one hand, the numerical methods such as the finite element method (FEM) [1] and the boundary element method (BEM). These methods are inevitably required to provide an accurate modelling of the sound propagation within a fully arbitrary geometry problem. On the other hand, the semi-analytical methods are broadly used for sound propagation in a slightly complex geometry configuration as efficient prediction tools. Among these semi-analytical methods, we can observe four different approaches. Firstly, the method of multiple scales used in the early 1980s by Rienstra [13] for sound propagation in cylindrical ducts with slowly varying liner properties. Secondly, the mode-matching method [8, 10] and point collocation one [9] based on the expansion of the sound field in terms of an orthogonal set of eigenfunctions of each specific problem. The difficulty that comes across this approach is the calculation of the complex eigenvalues for the lined segments. Root-finding routines are then required thus generating a time-consuming issue and a missing roots problem [6]. The third approach is the spectral or pseudo-spectra methods [5] which seek to interpolate the sound field in terms of general functions such as Chebyshev series. These methods converge exponentially, however their main drawback is the calculation of spurious modes. The fourth and the last semi-analytical approach is the multimodal method [11, 3]. The method presented in this paper is based on an improved multimodal method developed by Bi *et al.* [4, 2] for sound propagation in ducts with locally reacting lining. In the frame of this work, a lining consisting of porous material is considered. The effect of the complex properties of the liner cannot be described anymore by an effective impedance, since acoustic waves can propagate over it. For bulk reacting lining problems, the extension of the improved multimodal method is not trivial and the choice of the additionnal functions to improve the convergence of the method is a challenging task.

In this study, an efficient and reliable method is proposed for the propagation of sound in bidimensional bulk reacting lining ducts without flow. The lining consists of homogenous, isotropic or anisotropic, porous material with a perforate plate at the interface between the duct and the liner. The perforate plate is described mathematically by an impedance, assuming a continuous discontinuity along the interface. The extension of the proposed method to 3D rectangular cross section ducts is straightforward.

The improved multimodal method presented here is based on the sound field expansion in terms of the orthogonal and complete rigid duct modes *a priori* known. Additional functions that carry the informations relative to the boundary and continuity conditions of the problem are added to improve the convergence of the method. Those functions are chosen in order to ensure that the expansion may be derived term by term and converge rapidly to the exact solution. The convergence rate is improved from $O(n^{-2})$ to $O(n^{-4})$. This reduces significantly the computer memory requirement and the execution time. The rigid duct modes and the additional functions

*Electronic mail: lamiaa.kadiri.etu@univ-lemans.fr

are *a priori* known, thus calculations of the true liner modes are avoided. The eigenvalues are well-calculated including evanescent modes, and can be easily sorted as there are no spurious and no missing modes. All required eigenvalues are calculated by the method at a time and no initial guesses are needed. Furthermore, the wave propagation problem is considered as a scattering problem. Indeed, by matching the pressure and the axial velocity at the interface between the different lined segments, scattering matrices are obtained for each individual segment and then combined to construct a global scattering matrix for multiple segments. Different kind of sources can then be easily integrated without recalculating the scattering matrix.

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