Absorption of rigid frame porous grating with various shape rigid or soft inclusions

Benoit Nennig,[a)] Ygaäl Renou, Jean-Philippe Groby, and Yves Aurégan
Laboratoire d'Acoustique de l'Université du Maine, UMR6613 CNRS/Univ. du Maine, Avenue Olivier Messiaen, 72085 Le Mans Cedex 9, France

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The acoustic properties of a multilayer porous material backed by a rigid wall in which periodic inclusions are embedded are investigated by a mode matching approach. The proposed method is dedicated to Cartesian geometry problems for which the classical multipole decomposition, often used for grating studies, is not well adapted.
The inclusions may be of different types such as cavity filled with air or with another porous material or rigid scatterer of various shape. The influence of the inclusions is illustrated on various examples in order to enhance the absorption in the low frequency range or around specific frequencies.
These absorbing concepts as well as the computational approach are validated with experimental data obtained in anechoic room on rigid grating embedded in porous material made of 2 mm glass beads.

I. INTRODUCTION

Porous material are often used for their good sound absorbing capabilities in the middle and high frequency range. For a normal incidence excitation and a given material backed by a rigid wall, the low frequency performances are mainly limited by the treatment thickness. The thicker is the material, the better is the absorption. However for numerous practical applications, the thickness of the global structure is also submitted to integration constraints. Among all the solutions to enhance the compactness of the acoustics treatments there are the multilayer materials. Two main principles are in competition to optimize such material (i) the reduction of the impedance mismatch between the ambient fluid (the air) and the material to minimize specular reflection and obtain regular absorption curve on a large frequency band; (ii) the use of a highly resistive layer on the top to increase the excitation of volume resonances that induces highly absorption peaks at particular frequencies.

The purpose of this paper is to investigate an alternative to multilayer materials by studying the effects of embedded periodic inclusions in a porous layer[1–3]. The intuitive idea behind is to take advantage of the height and of the width of the treatment. To tackle this problem, a general method based on the mode matching method (MMM) is proposed. Thanks to the MMM scheme, it is now possible to investigate non circular shape inclusions. This approach seems to be a good compromise between speed-up, configuration diversity in comparison with finite element method. Moreover, there is no constraint between the size of the inhomogeneities and the wavelength in comparison with homogenization.

[a)] Electronic address: benoit.nennig@univ-lemans.fr

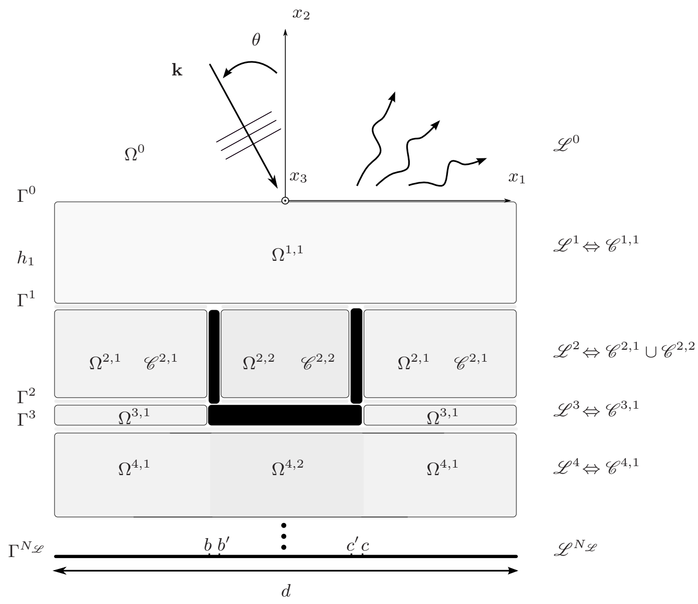

FIG. 1. Problem geometry.

II. FORMULATION OF THE METHOD

We consider here the absorption of an acoustic plane wave impinging at oblique incidence a d-periodic layered porous material (see Fig. 1). The $N_\mathscr{L}$ layers may contain some heterogeneities : (i) If the wave propagation is stopped by pairs of vertical rigid wall in a layer $\mathscr{L}^i$, the layer must be split, into $N^i_\mathscr{C}$ cells such $\mathscr{L}^i = \bigcup_{j=1}^{N^i_\mathscr{C}} \mathscr{C}^{i,j}$. (ii) Different materials $\Omega^{i,j}$ may be included in each cell. The interface Γ^i ($i = 0, \ldots, N_\mathscr{L}$) between two layers, may also be not homogeneous and may contain some rigid part.

In the following the skeleton of the porous material is considered as infinitely rigid, thus the Champoux-Allard-Johnson equivalent fluid model is used to get the equivalent bulk modulus $K^{i,j}$ and density $\rho^{i,j}$. The celerity is given by the ratio $c^{i,j} = \sqrt{K^{i,j}/\rho^{i,j}}$.

In the surrounding inviscid fluid, the sound speed and the density are respectively c^0 and ρ^0.

In each domain $\Omega^{i,j}$ (resp. in the surrounding fluid domain Ω^0), the equivalent fluid (resp. the fluid) satisfies the harmonic wave equation (the time dependence $\mathrm{e}^{-\mathrm{i}\omega t}$ is omitted for clarity)

$$\Delta p(\mathbf{x}) + (k^{i,j})^2 p(\mathbf{x}) = 0, \quad (1)$$

where the wave number is defined by $k^{i,j} = \omega / c^{i,j}$.

Since the geometry is periodic along x_1 and the excitation is due to a plane wave $p^{\mathrm{inc}} = A_0 \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}}$, each physical variable (call it X) satisfied the Floquet-Bloch relation

$$X(x_1 + d, x_2) = X(x_1, x_2)\mathrm{e}^{\mathrm{i}k_1 d}, \quad (2)$$

with the incident wave number $\mathbf{k} = (k_1, k_2) = k^0 \cdot (\sin\theta, -\cos\theta)$.

We are looking for the pressure field in the whole domain. As the problem is separable in each layer $\mathscr{L}^i$, the pressure can be written in a split form

$$p^i(\mathbf{x}) = \chi(x_1)\mathcal{Y}(x_2), \forall \mathbf{x} \in \mathscr{L}^i. \quad (3)$$

In each cell and in the surrounding fluid domain Ω^0, the eigenmodes satisfying the Floquet-Bloch condition are used as a decomposition basis along x_1. The x_2 direction correspond to the 'waveguide' axis and at each layers interface Γ^i ($i = 0, \ldots, N_{\mathscr{L}} - 1$), the continuity of the pressure and of the normal velocity apply. Then, on the rigid backing $\Gamma^{N_{\mathscr{L}}}$ the normal velocity must vanish. The modal expansion, are truncated to N in all the layers.

Following the configurations, two matching strategies have been used : (i) with soft inclusions such as air or another porous a classical matching[4] is suitable; (ii) with rigid inclusions an alternative procedure based on the re-expansion method[5] is more adapted because of the singular behavior of the velocity near corner. To overcome this problem, an expansion on function basis accounting for the velocity singularity near the corner is combined with the modal expansion. For this purpose, Gegenbauer orthogonal polynomial family are suitable because of their associated singular weighted function.

III. EXAMPLES

Numerical calculations of the absorption coefficient have been performed on a 2 cm × 2 cm elementary cell. For square highly resistive porous inclusions (see Fig. 2) and for rigid ⊔ inclusions (see Fig. 3) embedded in metal foam, the benefit of the inclusions can be clearly seen.

To the authors knowledges, this work is a first attempt to investigated the shape influence of rigid scatters embedded in a porous material. It has been shown that open shape such ⊔ inclusions are promising and better than massive shape[2] because less porous material is removed. Even if there is a need to explore new shapes or new simple shape combinations, these materials are already an alternative to multi-layering.

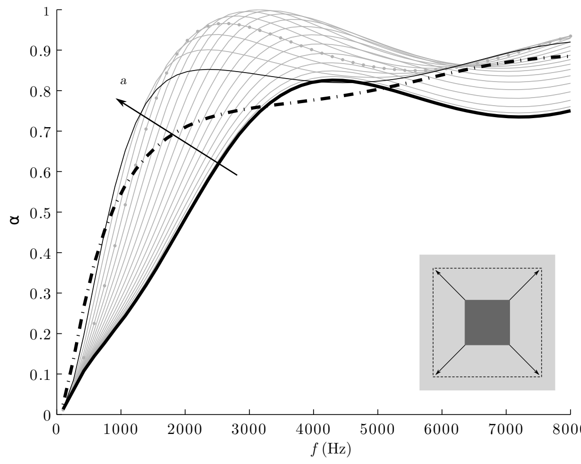

FIG. 2. Effect of the size of a square inclusion of a highly resistive porous material on the absorption coefficient. The size range from 1 to 18 mm (——) by 1 mm step. The reference homogeneous metal foam absorption is denoted by (━━) and the highly resistive porous material by (━·━).

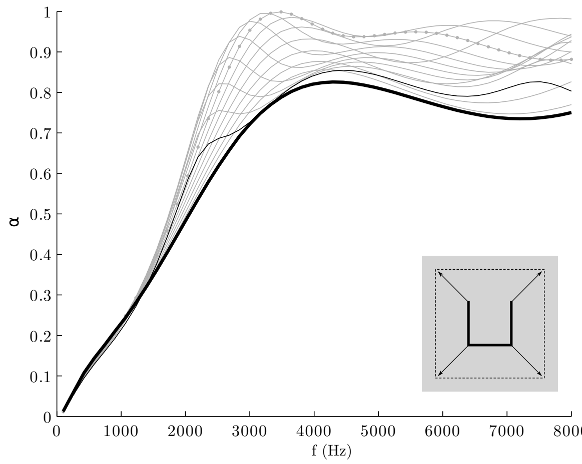

FIG. 3. Effect of the size of a rigid ⊔ inclusion on the absorption coefficient. The size range from 4 to 18 mm (——) by ≈1 mm step. The reference homogeneous metal foam absorption is denoted by (━━).

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