Sensitivity analysis of two “equivalent fluid” models

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Oral Presentation Prefered. No requirement for oral presentation.

Clogging is a phenomenon reducing significantly the performances of porous road surfaces in the draining off of rainwaters; it also affects the acoustic absorption of the road surface. Conversely, assessing the loss of acoustic absorption may be a way to access to the clogging progress in the road surface. The equivalent fluid models, which consider the frame as motionless and rigid, allow to connect microstructure related parameters (porosity, resistivity, etc) to measurable acoustic quantities (surface impedance, absorption coefficient) through quantities characterizing sound propagation in the medium (equivalent density ρ_{eq} and bulk modulus K_{eq} or characteristic impedance and wave number).

The present work is included in a work which aims at assessing structural evolution of a porous road pavement from an in situ measurement of the surface impedance. For materials such as road pavement, the motion of the frame may be neglected, and any of the equivalent fluid models may be used. This work is based on the inversion of a model of sund propagation in the materials from measurements results to access to geometric parameters. Analytical equations that may be used in inverse problems such as presented in [1] are of no use in this situation, as ρ_{eq} and K_{eq} are not available. The surface impedance measurement gives 2 equations, through its real and imaginary parts.

Models studied in this work are the so-called “Biot-Allard” model [2, 3, 4, 5] and the Johnson-Champoux-Allard model [6, 7], using respectively 3 parameters (open porosity Φ, static airflow resistivity σ and high frequency limit of dynamic tortuosity α_∞) and 5 parameters (Φ, σ, α_∞, and the viscous and thermal characteristic lengths Λ_v and Λ_t), to which one may add the thickness of the road wearing course when it is unknown. It is clear that the unknowns number exceeds the equations number.

However, road surfaces are very specific porous materials, especially because of their low porosity. This is why some of the geometric parameters required in the models may have a lower impact on the acoustic propagation quantities, or have an impact limited to a certain frequency range.

Therefore the purpose of this sensitivity analysis is to distinguish, for a typical porous road surface, the key parameters, the minor parameters, and to check if a model inversion may be realised thanks to available data.

Sensitivity index The sensitivity index used in this work has been defined by Sobol [8]. One considers a model with p input random variable X_i and one output random variable Y. Calculating the sensitivity index of Y to X_i consists in calculating a conditionnal variance: $V(E[Y|X_i])$. This quantity is divided by the global variance of Y to normalise the index. A way of estimating the sensitivity index has been given by Sobol [8] through a method based on Monte Carlo methods.

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Results 10^6-samples of each parameter have been taken in the following ranges, chosen from litterature [9, 10, 11]: $[0.08 - 0.22]$ for porosity, $[15000 - 85000] Nm^{-4}s$ for resistivity and $[1.3 - 2.7]$ for tortuosity; thickness has been set to 4 cm, which is common for a porous road surface.

Figure 1 shows the sensitivity indexes of surface impedance to each of the parameters of Biot-Allard model. One may see that no parameter can be neglected on $\Re(Z/Z_0)$. Concerning $\Im(Z/Z_0)$, at low frequencies , the sensitivity index (SI) of Φ is close to 1, and the SI of σ and α_∞ are close to 0.

The strong impact of porosity on imaginary part of surface impedance may also be proved using an asymptotic developpment; the sensitivity analysis allows to estimate the frequency range on which the low frequency approximation may be considered as valid.

Thus it seems that it would be possible to estimate porosity thanks to a measurement of $\Im(Z/Z_0)$ at low frequencies; by integrating this value of Φ in the set of equations $(\Re(Z/Z_0), \Im(Z/Z_0))$, it should be possible to estimate σ and α_∞, which means determining all parameters of Biot-Allard model.

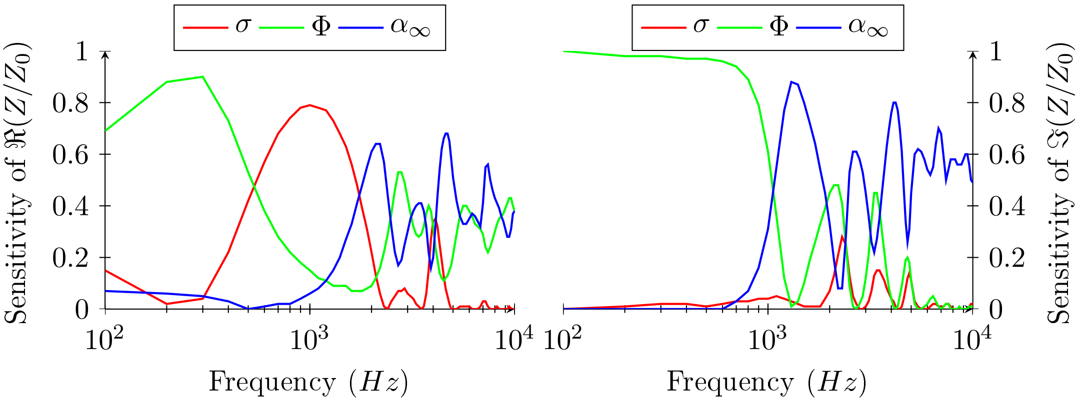

Figure 1: Sensitivity indexes of Biot-Allard model

As mentionned above, the thickness may be considered as an unknown of the model and integrated in the sensitivity analysis. Thickness is revealed as being the more important parameter for $\Im(Z/Z_0)$, being the only parameter that matters at high frequency. The only other parameter having a sensibility index higher than 0.1 is Φ on the range 100-1000 Hz. One might then determine thickness at high frequencies, integrating this value on the set of equations to get Φ at low frequency, and estimating the remaining parameters as said previously.

For Johnson-Champoux-Allard model, the same ranges are explored for Φ, Si and α_∞; for Λ_t the range is $[200; 2400]\mu m$ and for Λ_v $[60\mu m; \Lambda_t]$. Results are similar to those of Biot-Allard model (see figure 2): when carrying out the sensitivity analysis on $\Re(Z/Z_0)$, no parameter may be neglected or determined. On $\Im(Z/Z_0)$ on the contrary, Φ may still be determined from low frequencies, even if the SI of Λ_t is around 0.05, and α_∞ may be determined from high frequencies (from 6000 Hz). 3 unknown parameters remain: σ, Λ_v and Λ_t. It is possible to suppose a relation between Λ_v and Λ_t, such as $\Lambda_t = 2\Lambda_v$, to reduce the number of unknown parameters to 2, to be able to solve the system. The existence of this relation suppose a simple geometry of the material, wich is consistent with the use of Biot-Allard model.

As for Biot-Allard model, with the integration of thickness as a parameter, e is the more

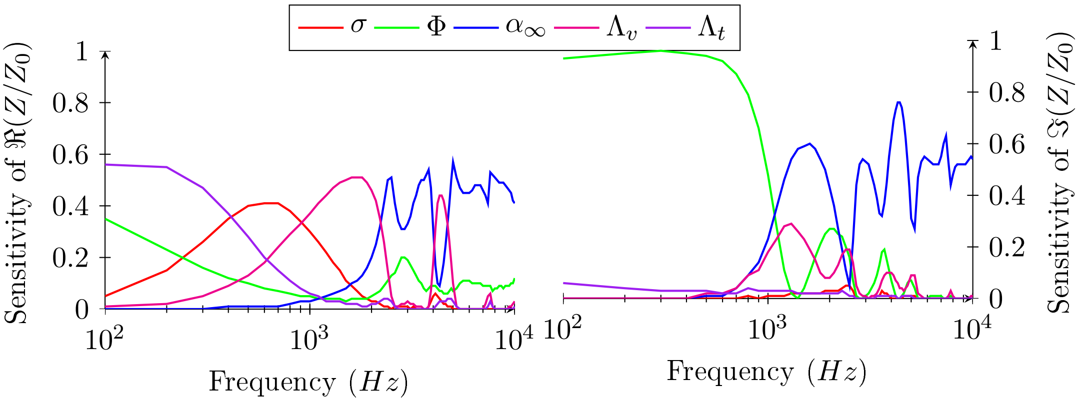

Figure 2: Sensitivity indexes of Johnson-Champoux-Allard model

important parameter for $\Im(Z/Z_0)$, on the whole frequency range, and the only parameter which matters after 2000 Hz, the second parameter being Φ before 1000 Hz. The same principle as for Biot-Allard model could then be applied.

Conclusion A sensitivity analysis of Biot-Allard and Johnson-Champoux-Allard models have been carried out, using a normalised index based on variance. Some of the results of this work were expectable, such as the high influence of Φ on $\Im(Z/Z_0)$ at low frequency, or the fact that α_∞ is more influent at high frequencies. In the intention of solving an inverse problem, it showed that when thickness is known, it seems that porosity might be determined at low frequency from $\Im(Z/Z_0)$, and tortuosity at high frequencies. When thickness is an unknown parameter of the model, this is the more important parameter on $\Im(Z/Z_0)$ and shoud then be determined first from $\Im(Z/Z_0)$ at high frequencies.

With the determination of porosity, the inverse problem might be solved for Biot-Allard model; for Johnson-Champoux-Allard it also requires the determination of tortuosity from $\Im(Z/Z_0)$ at high frequencies and to suppose the existence of a relation between Λ_v and Λ_t.

As the frequency ranges on which porosity and tortuosity or thickness need to be determined are unconnected and far from each other, measurements data need to be accurate in both low and high frequency ranges. Thus possibly two methods or tow configurations of the same method, may be used to collect this set of data, one optimal for low frequencies and one for high frequencies.

The perspectives of this work are to test to solve the inverse problem for Biot-Allard model, for which it might be done in a easier way when thickness is known.

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